Lecture Notes 1

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Preliminary

We begin with "Know Your Data" and "Show Your Data," to review some of the very initial components necessary for data analysis.

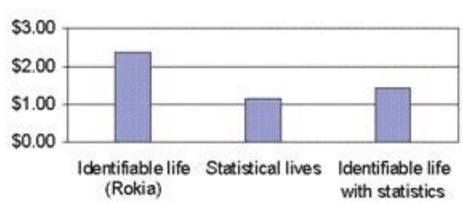
You might want to view online video 1; that covers similar basic information about measures of the data center such as mean, median, and mode; also measures of the spread of the data such as the standard deviation. Those notes are the middle part of this lecture. In class we will skip right to Lecture 2, where we apply these basic measures to learn about the PUMS dataset.

The Challenge

Humans are bad at statistics, we're just not wired to think this way. Despite – or maybe, because of this, statistical thinking is enormously powerful and it can quickly take over your life. Once you begin thinking like a statistician you will begin to see statistical applications to even your most mundane activities.

Not only are humans bad at statistics but statistics seem to interfere with essential human feelings such as compassion.

> "A study by Small, Loewenstein, and Slovic (2007) … gave people leaving a psychological experiment the opportunity to contribute up to $5 of their earnings to Save the Children. In one condition respondents were asked to donate money to feed an identified victim, a seven-year-old African girl named Rokia. They contributed more than twice the amount given by a second group asked to donate to the same organization working to save millions of Africans from hunger (see Figure 2). A third group was asked to donate to Rokia, but was also shown the larger statistical problem (millions in need) shown to the second group. Unfortunately, coupling the statistical realities with Rokia's story significantly reduced the contributions to Rokia.



> A follow-up experiment by Small et al. initially primed study participants either to feel ("Describe your feelings when you hear the word 'baby,'" and similar items) or to do simple arithmetic

calculations. Priming analytic thinking (calculation) reduced donations to the identifiable victim (Rokia) relative to the feeling-based thinking prime. Yet the two primes had no distinct effect on statistical victims, which is symptomatic of the difficulty in generating feelings for such victims." *(Paul Slovic, Psychic Numbing and Genocide, November 2007, Psychological Science Agenda, http://www.apa.org/science/psa/slovic.html)*

Yet although we're not naturally good at statistics, it is very important for us to get better. Consider all of the people who play the lottery or go to a casino, sacrificing their hard-earned money. (Statistics questions are often best illustrated by gambling problems, in fact the science was pushed along by questions about card games and dice games.)

Google, one of the world's most highly-regarded companies, famously uses statistics to guide even its smallest decisions:

> A designer, Jamie Divine, had picked out a blue that everyone on his team liked. But a product manager tested a different color with users and found they were more likely to click on the toolbar if it was painted a greener shade.
>
> As trivial as color choices might seem, clicks are a key part of Google's revenue stream, and anything that enhances clicks means more money. Mr. Divine's team resisted the greener hue, so Ms. Mayer split the difference by choosing a shade halfway between those of the two camps.
>
> Her decision was diplomatic, but it also amounted to relying on her gut rather than research. Since then, she said, she has asked her team to test the 41 gradations between the competing blues to see which ones consumers might prefer (Laura M Holson, "Putting a Bolder Face on Google" New York Times, Feb 28, 2009).

Substantial benefits arise once you learn stats. Specifically, if so many people are bad at it then gaining a skill in Statistics gives you a scarce ability – and, since Adam Smith, economists have known that scarcity brings value. (And you might find it fun!)

Leonard Mlodinow, in his book *The Drunkard's Walk*, attributes the fact that we humans are bad at statistics as due to our need to feel in control of our lives. We don't like to acknowledge that so much of the world is genuinely random and uncontrollable, that many of our successes and failures might be due to chance. When statisticians watch sports games, we don't believe sportscasters who discuss "that player just wanted it more" or other un-observable factors; we just believe that one team or the other got lucky.

As an example, suppose we were to have 1000 people toss coins in the air – those who get "heads" earn a dollar, and the game is repeated 10 times. It is likely that at least one person would flip "heads" all ten times. That person might start to believe, "Hey, I'm a good heads-tosser, I'm really good!" Somebody else is likely to have tossed "tails" ten times in a row – that person would probably be feeling stupid. But both are just lucky. And both have the same 50% chance of making "heads" on the next toss. Einstein famously said that he didn't like to believe that God played dice with the universe but many people look to the dice to see how God plays them.

Of course we struggle to exert control over our lives and hope that our particular choices can determine outcomes. But, as we begin to look at patterns of events due to many people's choices, then statistics become more powerful and more widely applicable. Consider a financial market: each individual trade may be the result of two people each analyzing the other's offers, trying to figure out how hard to press for a bargain, working through reams of data and making tons of calculations. But in aggregate, financial markets move randomly – if they did not then people could make a lot of money exploiting the patterns. Statistics help us both to see patterns in data that would otherwise see random and also to figure out when the patterns we observe are due to random chance. Statistics is an incredibly powerful tool.

Economics is a natural fit for statistical analysis since so much of our data is quantitative. Econometrics is the application of statistical analyses to economic problems. In the words of John Tukey, a legendary pioneer, we believe in the importance of "quantitative knowledge – a belief that most of the key questions in our world sooner or later demand answers to *by how much?* rather than merely to *in which direction?*"

This class

In my experience, too many statistics classes get off to a slow start because they build up gradually and systematically. That might not sound like a bad thing to you, but the problem is that you, the student, get answers to questions that you haven't yet asked. It can be more helpful to jump right in and then, as questions arise, to answer those at the appropriate time. So we'll spend a lot of time getting on the computer and actually doing statistics.

So the class will not always closely follow the textbook, particularly at the beginning. We will sometimes go in circles, first giving a simple answer but then returning to the most important questions for more study. The textbook proceeds gradually and systematically so you should read that to ensure that you've nailed down all of the details.

Statistics and econometrics are ultimately used for persuasion. First we want to persuade ourselves whether there is a relationship between some variables. Next we want to persuade other people whether there is such a relationship. Sometimes statistical theory can become quite Platonic in insisting that there is some ideal coefficient or relationship which can be discerned. In this class we will try to keep this sort of discussion to a minimum while keeping the "persuasion" rationale uppermost.

Step One: Know Your Data

The first step in any examination of data is to know that data – where did it come from? Who collected it? What is the sample of? What is being measured? Sometimes you'll find people who don't even know the units!

Economists often get figures in various units: levels, changes, percent changes (growth), log changes, annualized versions of each of those. We need to be careful and keep the differences all straight.

Annualized Data

At the simplest level, consider if some economic variable is reported to have changed by 100 in a particular quarter. As we make comparisons to previous changes, this is straightforward (was it more than 100 last quarter? Less?). But this has at least two possible meanings – only the footnotes or prior experience would tell the difference. It could imply that the actual change was 100, so if the item continued to change at that same rate throughout the year, it would change by 400 after 4 quarters. Or it could imply that the actual change was 25 and if the item continued to change at that same rate it would be 100 after 4 quarters – this is an annualized change. Most GDP figures are annualized. But you'd have to read the footnotes to make sure.

This distinction holds for growth rates as well. But annualizing growth rates is a bit more complicated than simply multiplying. (These are also distinct from year-on-year changes.)

CPI changes are usually reported as monthly changes (not annualized). GDP growth is usually annualized. So a 0.2% change in the month's CPI and a 2.4% growth in GDP are actually the same! Any data report released by a government statistical agency should carefully explain if any changes are annualized or "at an annual rate."

Seasonal adjustments are even more complicated, where growth rates might be reported as relative to previous averages. We won't yet get into that.

To annualize growth rates, we start from the original data (for now assume it's quarterly): suppose some economic series rose from 1000 in the first quarter to 1005 in the second quarter. This is a 0.5% growth from quarter to quarter (=0.005). To annualize that growth rate, we ask what would be the total growth, if the series continued to grow at that same rate for four quarters.

This would imply that in the third quarter the level would be $1005*(1 + 0.005) = 1005*(1.005) = 1000*(1.005)*(1.005) = 1000*(1.005)^2$; in the fourth quarter the level would be $1000*(1.005)*(1.005)*(1.005) = 1000*(1.005)^3$; and in the first quarter of next year the level would be 1000*(1.005) *(1.005) *(1.005) *(1.005) = $1000*(1.005)^4$, which is a little more than 2%.

This would mean that the annualized rate of growth (for an item reported quarterly) would be the final value minus the beginning value, divided by the beginning value, which is $\frac{1000(1.005)^4 - 1000}{1000} = (1.005)^4 - 1$.

Generalized, this means that quarterly growth is annualized by taking the single-quarter growth rate, g, and converting this to an annualized rate of $(1+g)^4 - 1$.

If this were monthly then the same sequence of logic would get us to insert a 12 instead of a 4 in the preceding formula. If the item is reported over t time periods, then the annualized rate is $(1+g)^t - 1$. (Daily rates could be calculated over 250 business days or 360 "banker's days" or 365/366 calendar days per year.)

The year-on-year growth rate is different. This looks back at the level from one year ago and finds the growth rate relative to that level.

Each method has its weaknesses. Annualizing needs the assumption that the growth could continue at that rate throughout the year – not always true (particularly in finance, where a stock could bounce by 1% in a day but it is unlikely to be up by over 250% in a year – there will be other large drops). Year-on-year changes can give a false impression of growth or decline after the change has stopped.

For example, if some item the first quarter of last year was 50, then it jumped to 60 in the second quarter, then stayed constant at 60 for the next two quarters, then the year-on-year change would be calculated as 20% growth even after the series had flattened.

Sometimes several measures are reported, so that interested readers can get the whole story. For examples, go to the US Economics & Statistics Administration, http://www.esa.doc.gov/, and read some of the "Indicators" that are released.

For example, on July 14, 2011, "The U.S. Census Bureau announced today that advance estimates of U.S. retail and food services sales for June, adjusted for seasonal variation and holiday and trading-day differences, but not for price changes, were $387.8 billion, an increase of 0.1 percent (±0.5%) from the previous month, and 8.1 percent (±0.7%) above June 2010." That tells you the level (not annualized), the monthly (not annualized) growth, and the year-on-year growth. The reader is to make her own inferences.

GDP estimates are annualized, though, so we can read statements like this, from the BEA's July 29 release, "Current-dollar GDP ... increased 3.7 percent, or $136.0 billion, in the second quarter to a level of $15,003.8 billion. " The figure, $15 trillion, is scaled to an annual GDP figure; we wouldn't multiply by 4. On the other hand, the monthly retail sales figures above **are not** multiplied by 12.

So if, for instance, we wanted to know the fraction of GDP that is retail sales, we could **NOT** divide 387.8/15003.8 = 2.6%! Instead either multiply the retail sales figure by 12 **or** divide the GDP figure by 12. This would get 31%. More pertinently, if we hear that government stimulus spending added $20 billion, we might want to try to figure out how much this helped the economy. Again, dividing 20/15003.8 = 0.13% (13 bps) but this is wrong! The $15tn is at an annual rate but the $20bn is not, so we've got to get the units consistent. Either multiply 50 by 4 or divide 15,003.8 by 4. (This mistake has been made by even very smart people!)

So don't make those foolish mistakes and know your data. If you have a sample, know what the sample is taken from. Often we use government data and just casually assume that, since the producers are professionals, that it's exactly what I want. But "what I want" is not always "what is in the definition." Much government data (we'll be using some of it for this class) is based on the Current Population Survey (CPS), which represents the civilian non-institutional population. Since it's the main source of data on unemployment rates, it makes good sense to exclude people in the military (who have little choice about whether to go to work today) or in prison (again, little choice). But you might forget this, and wonder why there are so few soldiers in the data that you're working with <*forehead slap!*>.

So know your data. Even if you're using internal company numbers, you've got to know what's being counted – when are sales booked? Warehouse numbers aren't usually quite the same as accounting numbers.

Show the Data

A hot field currently is "Data Visualization." This arises from two basic facts: 1. We're drowning in data; and 2. Humans have good eyes.

We're drowning in data because increasing computing power makes so much more available to us. Companies can now consider giving top executives a "dashboard" where, just like a driver can tell how fast the car is travelling right now, the executive can see how much profit is being made right now. Retailers have automated scanners at the cash register and at the receiving bay doors; each store can figure out what's selling.

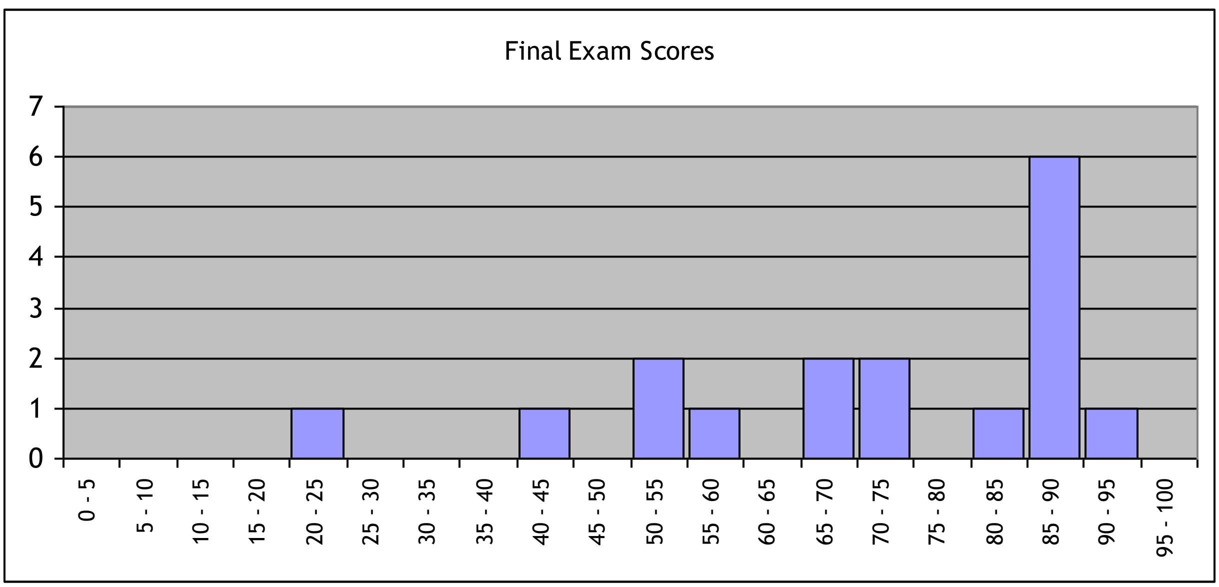
The data piles up while nobody's looking at it. An online store might generate data on the thousands of clicks simultaneously occurring, but it's probably just spooling onto some server's disk drive. It's just like spy agencies that harvest vast amounts of communications (voice, emails, videos, pictures) but then can't analyze them.

The hoped-for solution is to use our fundamental capacities to see patterns; convert machine data to visuals. Humans have good eyes; we evolved to live in the East African plains, watching all around ourselves to find prey or avoid danger. Modern people read a lot but that takes just a small fraction of the eye's nerves; the rest are peripheral vision. We want to make full use of our input devices.

But putting data into visual form is really tough to do well! The textbook has many examples to help you make better charts. Read Chapter 3 carefully. The homework will ask you to try your hand at it.

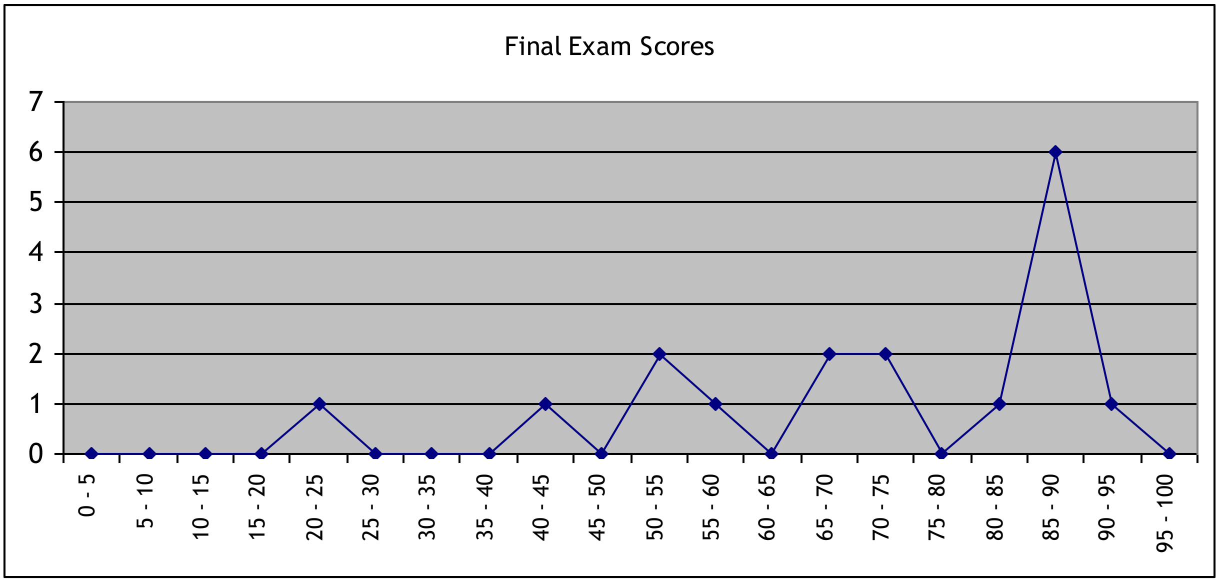
Histograms

You might have forgotten about histograms. A histogram shows the number (or fraction) of outcomes which fall into a particular bin. For example, here is a histogram of scores on the final exam for a class that I taught:



This histogram shows a great deal of information; more than just a single number could tell. (Although this histogram, with so many one- or two-step sizes, could be made much better.)

Often a histogram is presented, as above, with blocks but it can just as easily be connected lines, like this:

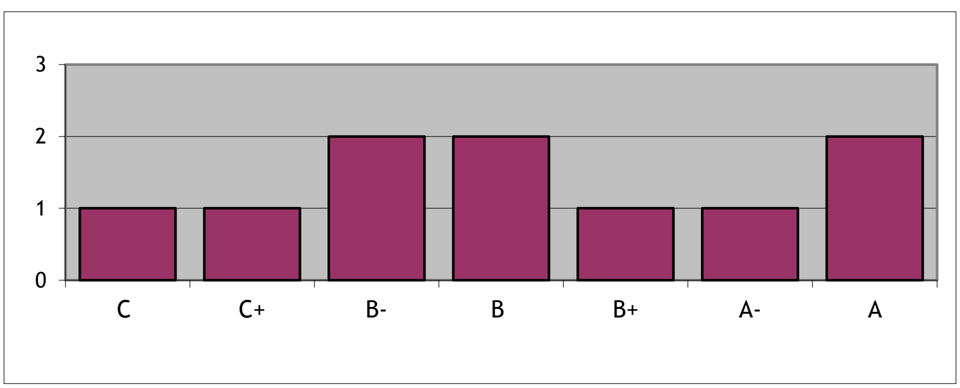


The information in the two charts is identical.

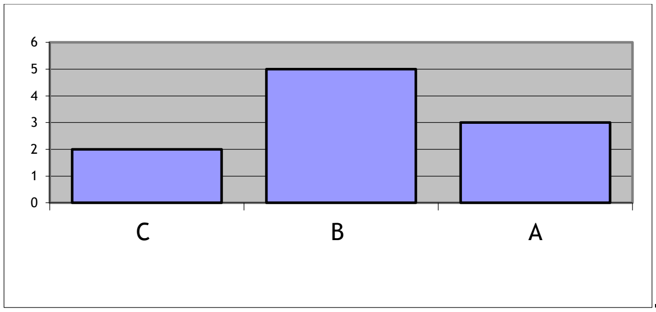
Histograms are a good way of showing how the data vary around the middle. This information about the spread of outcomes around the center is very important to most human decisions – we usually don't like risk.

Note that the choice of horizontal scaling or the number of bins can be fraught.

For example consider a histogram of a student's grades. If we leave in the A- and B+ grades, we would show a histogram like this:



whereas by collapsing together the grades into A, B, and C categories we would get something more intelligible, like this:



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This shows the central tendency much better – the student has gotten many B grades and slightly more A grades than C grades. The previous histogram had too many categories so it was difficult to see a pattern.

Basic Concepts: Find the Center of the Data

You need to know how to calculate an average (mean), median, and mode. After that, we will move on to how to calculate measures of the spread of data around the middle, its variation.

Average

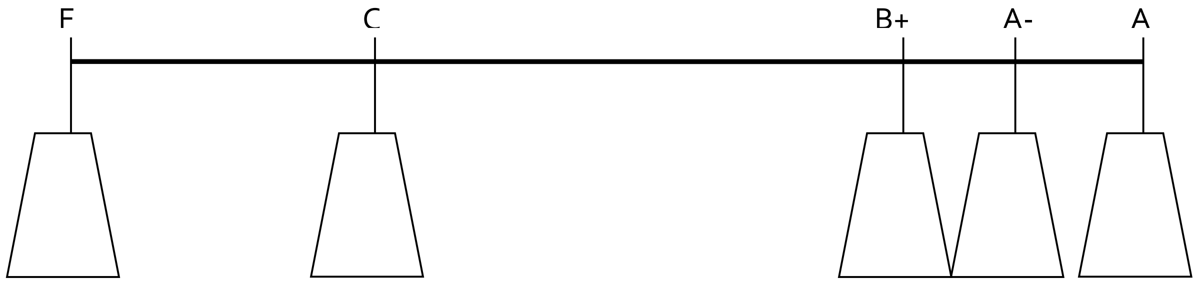
There are a few basic calculations that we start with. You need to be able to calculate an average, sometimes called the mean.

The average of some values, X, when there are N of them, is the sum of each of the values (index them by i) divided by N, so the average of X, sometimes denoted $\bar{X}$, is

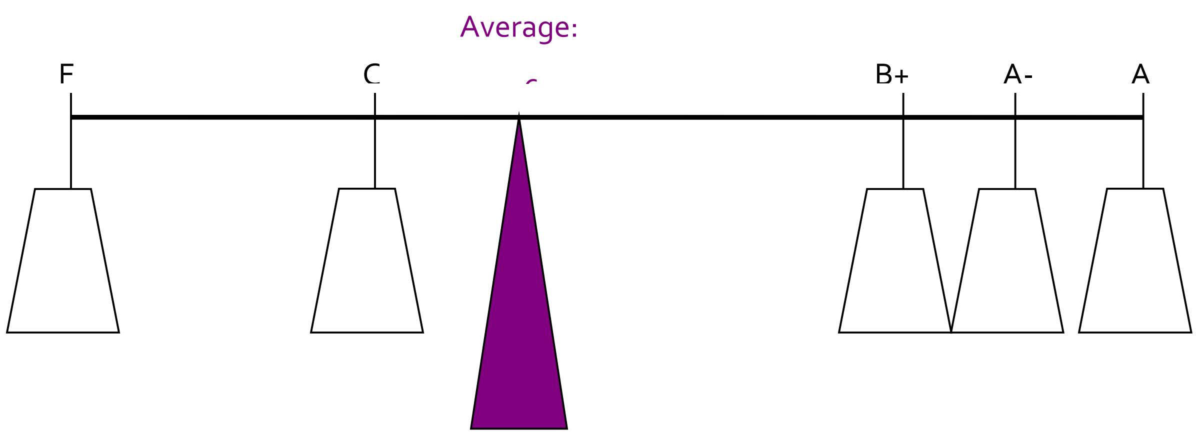
$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \ .$$

The average value of a sample is NOT NECESSARILY REPRESENTATIVE of what actually happens. There are many jokes about the average statistician who has 2.3 kids. If there are 100 employees at a company, one of whom gets a $100,000 bonus, then the average bonus was $1000 – but 99 out of 100 employees didn't get anything.

A common graphical interpretation of an average value is to interpret the values as lengths along which weights are hung on a see-saw. The average value is where a fulcrum would just balance the weights. Suppose a student is calculating her GPA. She has an A (worth 4.0), an A- (3.67), a B+ (3.33), a C (2.0) and one F (0) [she's having troubles!]. We could picture these as weights:



The weights "balance" at the average point (where (0 + 2 + 3.33 + 3.67 + 4)/5 = 2.6):



So the "bonus" example would look like this, with one person getting $100,000 while the other 99 get nothing:

Where there are actually 99 weights at "zero." But even one person with such a long moment arm can still shift the center of gravity away.

Bottom Line: The average is *often* a good way of understanding what happens to people within some group. But it is *not always* a good way.

Sometimes we calculate a weighted average using some set of weights, w, so

$$X_{weighted\ Average} = \sum_{i=1}^{n} w_i X_i \text{, where } \sum_{i=1}^{n} w_i = 1 .$$

Your GPA, for example, weights the grades by the credits in the course. Suppose you get a B grade (a 3.0 grade) in a 4-credit course and an A- grade (a 3.67 grade) in a 3-credit course; you'd calculate GPA by multiplying the grade times the credit, summing this, then dividing by the total credits:

$$GPA = \frac{3 \cdot 4 + 3.67 \cdot 3}{4+3} = \frac{4}{4+3} 3 + \frac{3}{4+3} 3.67 = 3.287 .$$

So in this example the weights are $w_1 = \frac{4}{4+3}, w_2 = \frac{3}{4+3}$.

When an average is projected forward it is sometimes called the "Expected Value" where it is the average value of the predictions (where outcomes with a greater likelihood get greater weight). This nomenclature causes even more problems since, again, the "Expected Value" is NOT NECESSARILY REPRESENTATIVE of what actually happens.

To simplify some models of Climate Change, if there is a 10% chance of a 10° increase in temperature and a 90% chance of no change, then the calculated Expected Value is a 1° change – but, again, this value does not actually occur in any of the model forecasts.

For those of you who have taken calculus, you might find these formulas reminiscent of integrals – good for you! But we won't cover that now. But if you think of the integral as being just an extreme form o f a summation, then the formula has the same format.

Median

The median is another measure of what happens to a 'typical' person in a group; like the mean it has its limitations. The median is the value that occurs in the 50th percentile, to the person (or occurrence) exactly in the middle. If there are an odd number of outcomes, otherwise it is between the two middle ones.

In the bonus example above, where one person out of 100 gets a $100,000 bonus, the median bonus is $0. The two statistics combined, that the average is $1000 but the median is zero, can provide a better understanding of what is happening. (Of course, in this very simple case, it is easiest to just say that one person got a big bonus and everyone else got nothing. But there may be other cases that aren't quite so extreme but still are skewed.)

Mode

The mode is the most common outcome; often there may be more than one. If there were a slightly more complicated payroll case, where 49 of the employees got zero bonus, 47 got $1000, and four got $13,250 each, the mean is the same at $1,000, the median is now equal to the mean [review those calculations for yourself!], but the mode is zero. So that gives us additional information beyond the mean or median.

Spread around the center

Data distributions differ not only in the location of their center but also in how much spread or variation there is around that center point. For example a new drug might promise an average of 25% better results than its competitor, but does this mean that 25% of patients improved by 100%, or does this mean that everybody got 25% better? It's not clear from just the central tendency. But if you're the one who's sick, you want to know.

This is a familiar concept in economics where we commonly assume that investors make a tradeoff between risk and return. Two hedge funds might both have a record of 10% returns, but a record of 9.5%, 10%, and 10.5% is very different from a record of 0%, 10%, and 20%. (Actually a record of always winning, no matter what, distinguished Bernie Madoff's fund…)

You might think to just take the average difference of how far observations are from the average, but this won't work.

There's an old joke about the tenant who complains to the super that in winter his apartment is 50° and in summer is 90° -- and the super responds, "Why are you complaining? The apartment is a comfortable 70° on average!" (So the tenant replies "*I'm complaining because I have a squared error loss function*!" If you thought that was funny, you're a stats geek already!)

The average deviation from the average is always zero. Write out the formulas to see.

The average of some N values, $X_1, X_2, \ldots X_N$, is given by $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$.

So what is the average deviation from the average, $\sum_{i=1}^{N}\left(X_i - \bar{X}\right)$?

We know that $\sum_{i=1}^{N}\left(X_i - \bar{X}\right) = \sum_{i=1}^{N} X_i - \sum_{i=1}^{N} \bar{X}$ and, since $\bar{X}$ is the same for every observation, $\sum_{i=1}^{N} \bar{X} = N\bar{X} = \sum_{i=1}^{N} X_i$, if we substitute back from the definition of $\bar{X}$. So $\sum_{i=1}^{N}\left(X_i - \bar{X}\right) = 0$. We can't re-use the average. So we want to find some useful, sensible function [or functions], $f(\cdot)$, such that $\sum_{i=1}^{N} f\left(X_i - \bar{X}\right) \neq 0$.

Standard Deviation

The most commonly reported measure of spread around the center is the standard deviation. This looks complicated since it squares the deviations and then takes the square root, but is actually quite generally useful.

The formula for the standard deviation is a bit more complicated:

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2} .$$

Before you start to panic, let's go through it slowly. First we want to see how far each observation is from the mean,

$$\left(X_i - \bar{X}\right).$$

If we were to just sum up these terms, we'd get nothing – the positive errors and negative errors would cancel out.

So we square the deviations and get

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 ,$$

and then just divide by n to find the average squared error, which is known as the variance, which is

$$\sigma_X^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2 .$$

The standard deviation is the square root of the variance; $\sigma_X = \sqrt{\sigma_X^2}$ $= \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2}$.

Of course you're asking why we bother to square all of the parts inside the summation, if we're only going to take the square root afterwards. It's worthwhile to understand the rationale since similar questions will re-occur. The point of the squared errors is that they don't cancel out. The variance can be thought of as the average size of the squared distances from the mean. Then the square root makes this into sensible units.

The variance and standard deviation of the population divides by N; the variance and standard deviation of a sample divide by (N – 1). This is referred to as a "degrees of freedom correction," referring to the fact that a sample, after calculating the mean, has lost one "degree of freedom," so the standard deviation has only (N – df) remaining. You could worry about that difference or you could note that, for most datasets with huge N (like the ATUS with almost 100,000), the difference is too tiny to worry about.

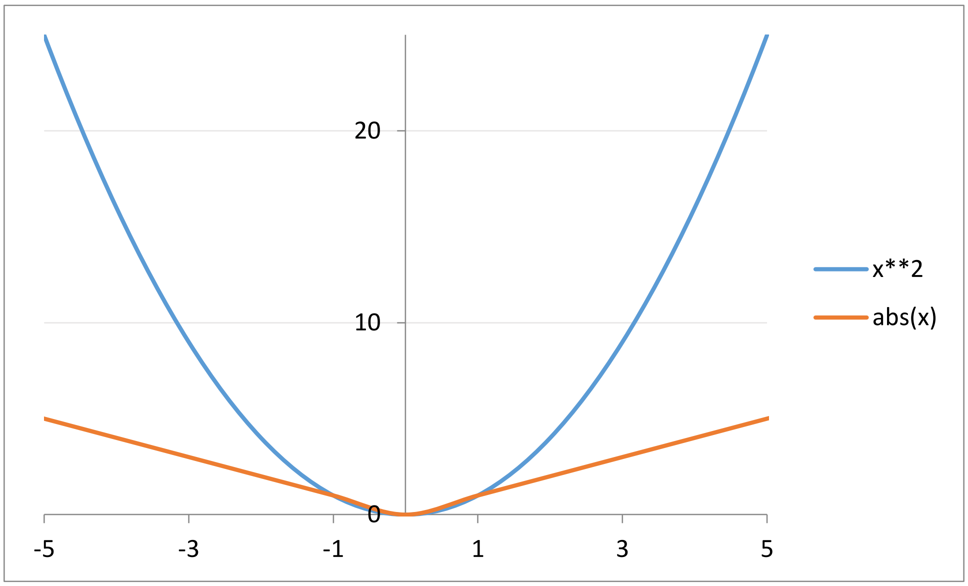
Our notation generally uses Greek letters to denote population values and English letters for sample values, so we have

$$s_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$ and

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2} .$$

As you learn more statistics you will see that the standard deviation appears quite often. Hopefully you will begin to get used to it.

We could look at other functions of the distance of the data from the central measure, $f(\cdot)$, such that $\sum_{i=1}^{N} f\left(X_i - \bar{X}\right) \neq 0$ -- for example, the mean of the absolute value, $\frac{1}{N}\sum_{i=1}^{N}\left|X_i - \bar{X}\right|$. By recalling the graphs of these two functions you can begin to appreciate how they differ:



So that squaring the difference counts large deviations very much worse than small deviations, whereas an absolute deviation does not. So if you're trying to hit a central target, it might well make sense that wider and wider misses should be penalized worse, while tiny misses should be hardly counted.

There is a relationship between the distance measure selected and the central parameter. For example, suppose I want to find some number, Z, that minimizes a measure of distance of this number, Z, from each observations. So I want to minimize $\frac{1}{N}\sum_{i=1}^{N} f(X_i - Z)$. If we were to use the absolute value function then setting Z to the median would minimize the distance. If we use instead the squared function then setting Z to the average would minimize the distance. So there is an important connection between the average and the standard deviation, just as there is a connection between the median and the absolute deviation. *(Can you think of what distance measure is connected with the mode?)*

If you know calculus, you will understand why, in the age before computer calculations, statisticians preferred the squared difference to the absolute value of the difference. If we look for an estimator that will minimize that distance, then in general in order to minimize something we will take its derivative. But the derivative of the absolute value is undefined at zero, while the squared distance has a well-defined derivative.

Sometimes you will see other measures of variation; the textbook goes through these comprehensively. Note that the Coefficient of Variation, $\frac{s}{\overline{X}}$, is the reciprocal of the signal-to-noise ratio. This is an important measure when there is no natural or physical measure, for example a Likert scale. If you ask people to rate beers on a scale of 1-10 and find that consumers prefer Stone's Ruination Ale to Budweiser by 2 points, you have no idea whether 2 is a big or a small difference – unless you know how much variation there was in the data (i.e. the standard deviation). On the other hand, if Ruination costs $2 more than Bud, you can interpret that even without a standard deviation.

In finance, this signal/noise ratio is referred to as the Sharpe Ratio, $\frac{\overline{R}-r_f}{\sigma}$, where $\overline{R}$ are the average returns on a portfolio and r_f is the risk-free rate; the Sharpe Ratio tells the returns relative to risk.

Sometimes we will use "Standardized Data," usually denoted as Z_i, where the mean is subtracted and then we divide by the standard deviation, so $Z_i = \frac{X_i - \overline{X}}{s}$. This is interpretable as measuring how many standard deviations from the mean is any particular observation. This allows us to abstract from the particular units of the data (meters or feet; Celsius or Fahrenheit; whatever) and just think of them as generic numbers.

Now Do It!

We'll use data from the Census PUMS, on just people in New York City, to begin actually doing statistics using the analysis program called SPSS. There are further lecture notes on each of those topics. Read those carefully; you'll need them to do the homework assignment.

Overview of PUMS

We will use data from the Census Bureau's "Public Use Microdata Survey," or PUMS. This is collected in the American Community Survey; just about every ten years since 1790 the Census has made a complete enumeration of the US population as required by the Constitution.

We will work on this data using SPSS. Later I give an overview of the basics of how to use that program (there are also videos online).

The dataset, which is only just information on respondents in the five boroughs of New York City, is ready to use in SPSS. Download it from the class web page (or InYourClass page) onto your computer desktop. It is zipped so you must unzip it. Remember that if you're in the computer lab, just double-clicking on the SPSS file may not automatically start up SPSS; you'll get some error code. So use the Start bar to find SPSS and start the program that way. Then open up your dataset once the program has loaded.

SPSS has two views of the dataset: Variable View and Data View. Usually we use the Variable View; this lists all of the different information available.

The dataset has information on 315,771 people in 133,043 households. If there is a family living together in an apartment, say a mother and two kids, then each person has a row of data telling about him/her (age, gender, education, etc) but only the head of household (in this case, the mother) would have information about the household (how much is spent on rent, utilities, etc.). Depending on what analysis is to be made, the researcher might want to look at all the people or all of the households (or subsets of either). If you look at the "Data View" tab you can see the difference. (Note that the "head of household" is defined by the person interviewed so it could be the man or woman, if there are both.)

The first column of data is a serial number, shared by each person in the household. After that you can see that some variables are filled in for every person (age, female, education levels) but other variables are only filled in for one person in the household (has_kids, kids_under6, kids_under17).

Many of the variables are coded as "dummy variables" which simply means that they have a value of zero or one – a one codes as "yes, true" and a zero is "no, false." So one of the first dummy variables is named "female" and women have a 1 while men have a 0. (According to the government, everybody must be one or the other.)

There are variables coding people's race/ethnicity, if they were born in the US or a foreign country, how much schooling they have, if they are single or married, if they're a veteran (and when they served), even what borough they live in and how they commute to work. There is some greater detail about ancestry (where people can write in detail about their background). There is information about their incomes. For the household there is information about the dwelling including when built, number of various rooms, how recently they moved, amount paid for fuel, mortgage/rent and fraction of monthly household income that is spent on mortgage/rent, etc.

Basics of government race/ethnicity classification

The US government asks questions about people's race and ethnicity. These categories are social constructs, which is a fancy way of pointing out that they are not based on hard science but on people's own views of themselves (influenced by how people think that other people think of them...). Currently the standard classification asks people separately about their "race" and "ethnicity" where people can pick labels from each category in any combination.

The "race" categories that are listed on the government's form are: "White alone," "Black or African-American alone," "American Indian alone," "Alaska Native alone," "American Indian and Alaska Native tribes specified; or American Indian or Alaska native, not specified and no other race," "Asian alone," "Native Hawaiian and other Pacific Islander alone," "Some other race alone," or "Two or more major race groups." (Then the supplemental race categories offer more detail.)

These are a peculiar combination of very general (well over 40% of the world's population is "Asian") and very specific ("Alaska Native alone") representing a peculiar history of popular attitudes in the US. Only in the 2000 Census did they start to classify people in mixed races. If you were to go back to historical US Censuses from more than a century ago, you would find that the category "race" included separate entries for Irish and French and various other nationalities. Stephen J Gould has a fascinating book, *The Mismeasure of Man*, discussing how early scientific classifications of humans tried to "prove" which nationalities/races/groups were the smartest.

Note that "Hispanic" is not "race" but rather ethnicity (includes various other labels such as Spanish, Latino, etc.). So a respondent could choose "Hispanic" and any race category – some choose "White," some choose "Black," some might be combined with any other of those complicated racial categories.

What that means, specifically for us reporting statistics on a dataset like this, is that we can easily find that, of the 315,771 people in the PUMS dataset who live in the five boroughs of New York City, 48.2% report their race as "White alone" and 24.2% as "Black alone," 12.3% report as "Asian alone," 12.7% report "some other race alone," 2.2% report multiple races, and less than 1% report any Native American category. Then 24.1% classify their ethnicity as Hispanic. Can we just take the 48.2% White, subtract the 24.1% Hispanic to say that 24.1% are "non-Hispanic White" (a category commonly used in other government classifications)? NO! Because that assumes that all of the people who self-classified as Hispanic were also self-classified as "White only" which is not true. We would have to create a new variable for non-Hispanic White to find that proportion. (Below I'll explain how to do this with SPSS.)

The Census Bureau gives more information here, http://www.census.gov/newsroom/minority_links/minority_links.html

All of these racial categories makes some people uneasy: is the government encouraging racism by recognizing these classifications? Some other governments choose not to collect race data. But that doesn't mean that there are no differences, only that the government doesn't choose to measure any of these differences. In the US, government agencies such as the Census and BLS don't generally collect data on religion (except, for historical reasons, Judaism, sometimes considered a race or ethnicity – none of this makes any logical sense!).

About SPSS

SPSS is a popular and widely-used statistical program. It is powerful but not too overwhelming for a beginner. SPSS is a bit harder than Excel but gives you a much wider menu of statistical analysis. You don't have to write computer programs like some of the others – you can just use drop-down menus and point and click.

Why learn this particular program? You should not be monolingual in statistical analysis, it is always useful to learn more programs. The simplest is Excel, which is very widely used but has a number of limitations – mainly that, in order to make it easy for ordinary people to use, they made it tough for power users. SPSS is the next step: more powerful but also a bit more difficult. Next is Stata and SAS, which are a bit more powerful but also tougher to use. Matlab is great but requires writing programs of computer code. R is an open-source version (we'll use that a little for this class) that is used by many researchers but it requires some work to learn. Python is necessary if you're going to become a real data analyst. The college has SPSS, SAS, and Matlab freely available in all of the computer labs.

You might be tempted to just use Excel; resist! Excel doesn't do many of the more complex statistical analyses that we'll be learning later in the course. Make the investment to learn a better program; it has a very good cost/benefit ratio.

The Absolute Beginning

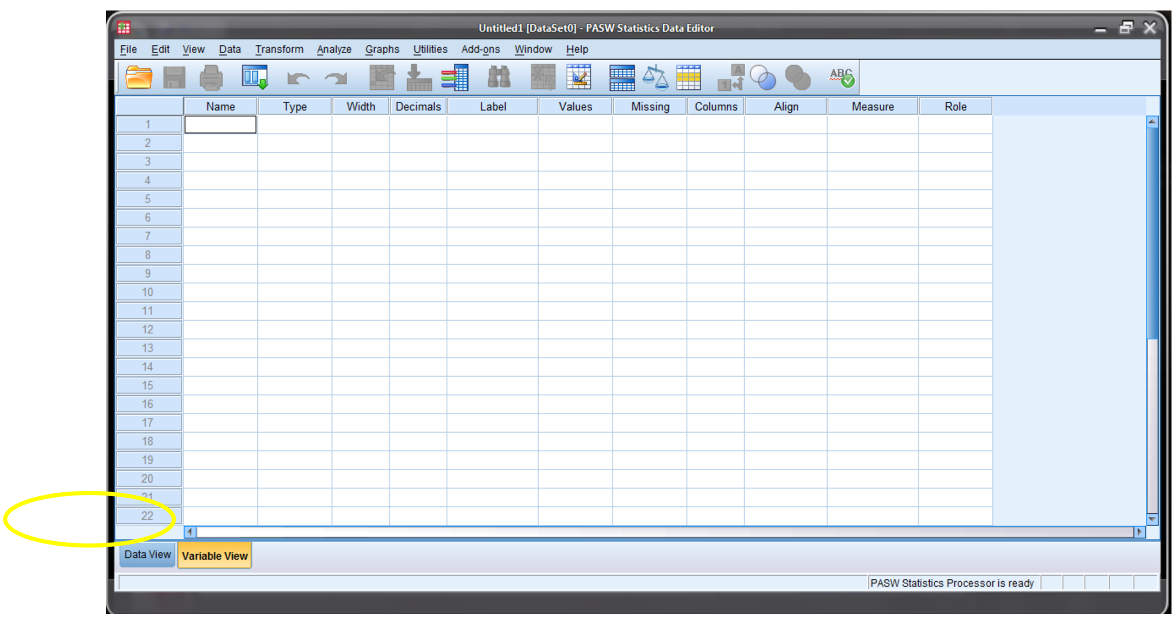
Start up SPSS. On any of the computers in the Economics lab (6/150) double-click on the "SPSS" logo on the desktop to start up the program. In other computer labs you might have to do a bit more hunting to find SPSS (if there's no link on the desktop, then click the "Start" button in the lower left-hand corner, and look at the list of "Programs" to find SPSS).

Sometimes double-clicking on a file that is associated with SPSS **doesn't** work! Especially if it's zipped. Same if you try to download a file and automatically start up SPSS. So start SPSS from the Start bar or desktop icon.

SPSS usually brings up a screen like the one below asking "`What would you like to do?`" which offers some shortcuts. Just "Cancel" this screen if it appears (later, as you get more familiar with the program, you might find those shortcuts more useful).

Load a SPSS Dataset

When SPSS starts, you will be in the "`SPSS Data Editor`" which looks like this.



Click on "`File`" then choose "`Open`" then "`Data…`" [**not** "`File/Open Database`" – that's different].

To open the ATUS data, download it from the class webpage onto your computer desktop. Start SPSS. Then "`File \ Open \ Data...`" and find "`ATUS_2003-09.sav`". (Many datasets are zipped, you first unzip it, then load it.)

SPSS has two tabs (at the bottom left, in the yellow circle above) to change the way you view your data. The "Data View" tab shows the data the way it would look if it were on an Excel sheet. The "Variable View" tab shows more information on the particular variable – most importantly, the "`name`", "`label`", and "`values`". The *Name* is how SPSS refers to the variable in its menus – these names tend to be inscrutable but you can think of them as nicknames. The *Label* gives more details, so use the mouse to expand that column so that you can read more. Then *values* tells you useful information about how the variable is coded.

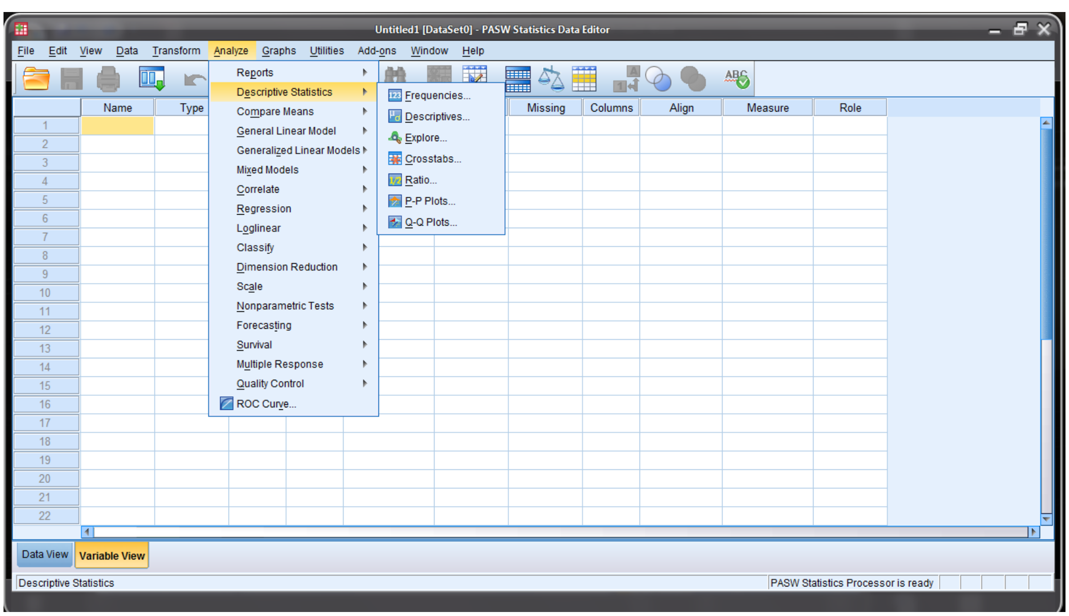
Save your Work!

After you've made changes, you don't want to lose them and have to re-do them. So save your dataset! ("`File`" then "`Save`") You might want to give it a new name every so often, so that you can easily revert back to an old version if you really screw up on some day.

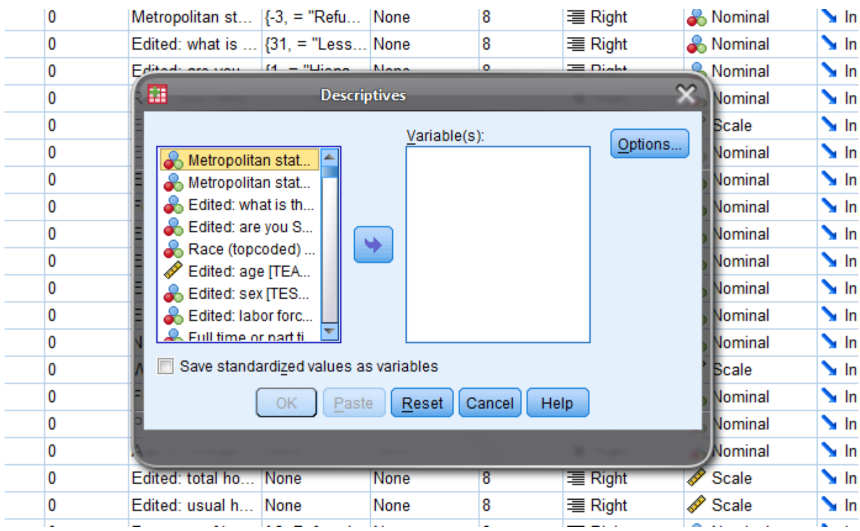
The computers in the lab wipe the memory clean when you log off so back up your data. Either online (email it to yourself or upload to Google Drive or iCloud or Blackboard) or use a USB drive. Also, figure out how to "zip" your files (right-click on the data file) to save yourself some hours of up/download time…

Getting Basic Statistics

From either the "Data View" or "Variable View" tab, click "`Analyze`" then "`Descriptive Statistics`" then "`Descriptives`":



This will bring up a dialog box asking you which variables you want to get Descriptive Statistics on.



Click on the variable you want. Then click the arrow button in the center box, which will move the variables into the column labeled "`Variable(s)`". If you make a mistake and move the wrong variable, just highlight it in the "`Variable(s)`" column and use the arrow to move it back to the left.

Then click "`OK`" and let the computer work.

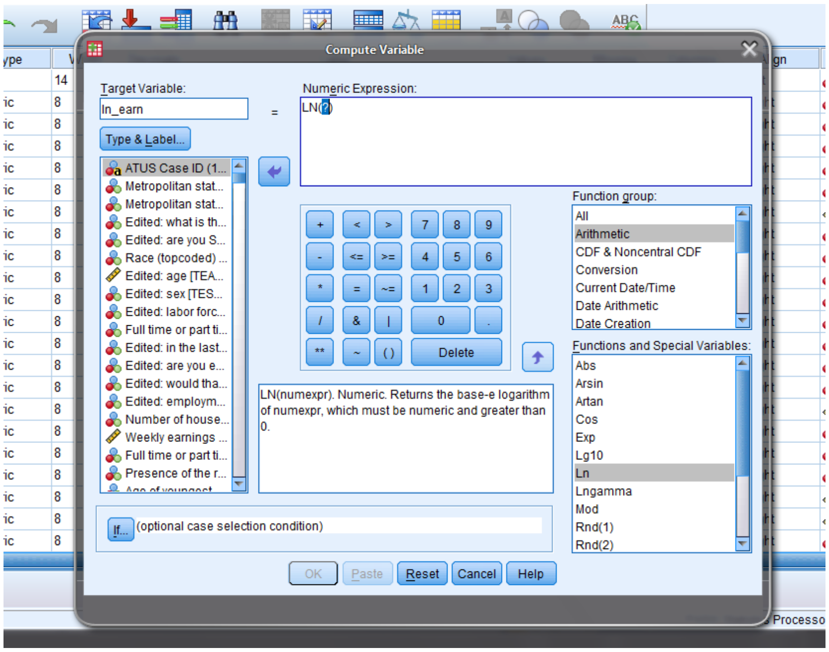
If you want a bunch that are all together in the list, click on the first variable that you want, then hold down the "`Shift`" key and click on the last variable -- this highlights them all. If you want a bunch that are separated, hold down the "`Ctrl`" key and click on the ones you want.

Later, once you're feeling confident, click on "Options" to see what's there.

Create New Variables, like Age-squared or Interaction Age*Dummy, or take logs or whatever

We often create new variables. One common transformation is taking the log. This is a common procedure to cut down the noise and help to examine growth trends. Click on "`Transform`" and then "`Compute...`". This will bring up a dialog box labeled "`Compute Variable`".

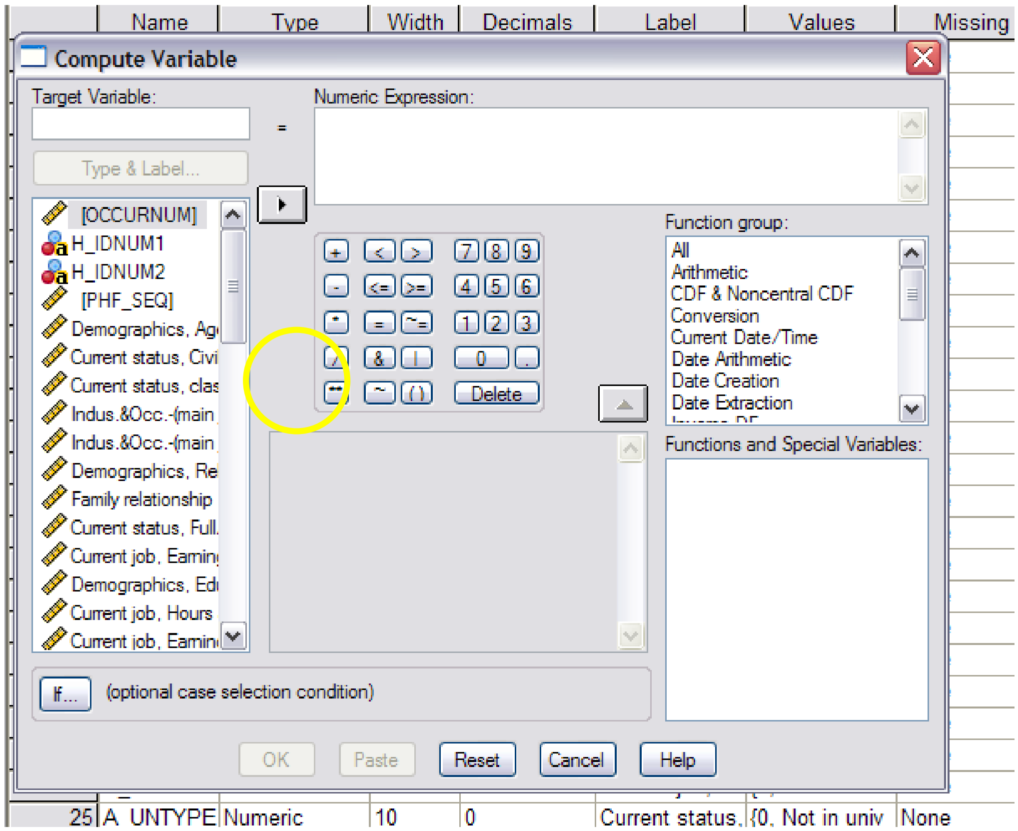
Type in the new variable name (whatever you want, just remember it!) under "`Target Variable`". (You can click '`Type & Label`" if you want to enter more info that can remind yourself later.) For example we'll find the log (natural log) of weekly earnings.



Under "`Target Variable`" type in the new name, "`ln_earn`" or whatever and then in "`Numeric Expression`" you tell it what this new variable is. You can make any complicated or convoluted functions that are necessary for particular analyses; for now find the "`Function Group`" to click on "`Arithmetic`" and then in the "`Functions and Special Variables`" list below find "`Ln`". Double-click it and see that SPSS puts it up into the "`Numeric Expression`" box with a `(?)` in the argument. Double-click on the variable, weekly earnings (`TRERNWA`), that you want to use and then hit "OK".

You'll get a bunch of errors where the program complains about trying to find the log of zero, but it still does what you need. For wages, where many people have wage=0, we often use lnwage = ln(wage + 1) which eliminates the problem of ln(0) that returns an error; for most other people the distinction between ln(1000) and ln(1001) is tiny. You can go back and re-do your variable if you're feeling a need to be tidy.

We often recode using logical (Boolean) algebra, so for example to make a variable "Hispanic" you'd type "`Hispanic`" into the Target Variable, then click the "`( )`" button (see the yellow circle in the screenshot below) to get a parenthesis, double-click the variable that codes ethnicity so as to get `PEHSPNON` in the "`Numeric Expression`" and then add "`=1`" to finish, so getting a relationship that Hispanic is defined as: (`PEHSPNON = 1`). SPSS understands that whenever that relation is true, it will put in a 1; where false it will put in a 0.



There are other logical buttons (also in the yellow circle above) for putting together various logical statements. The line up and down, `|`, represents the logical "OR"; the tilde, `~`, is logical "NOT".

If you wanted to create a variable for those who report themselves as African-American and Hispanic, you'd create the expression `(AfricanAmerican = 1) & (Hispanic = 1)`.

If we want more combinations of variables then we create those. Usually a statistical analysis spends a lot of time doing this sort of housekeeping – dull but necessary.

Re-Coding complicated variables (like race, education, etc) from initial data

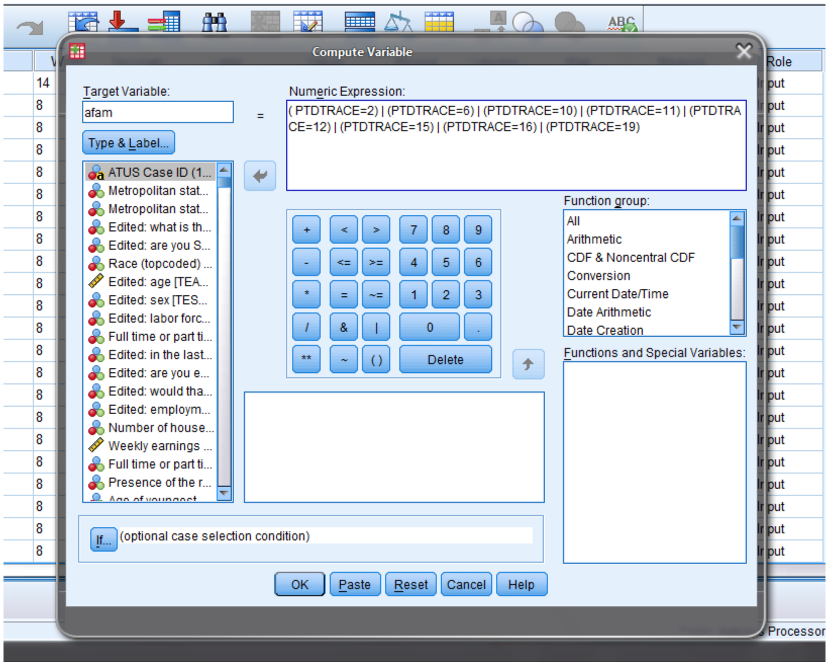
Often we have more complicated variables so we need to be careful in considering the "`Values`" labels. For instance in the ATUS, as you look at the "`Variable View`" of your dataset, one of the first variables in the dataset has the name "`PEEDUCA`", which is short for "PErson EDUCation Achieved" – the person's education level. But the coding is strange: under "`Values`" you should see a box with "..." in it – click on that to see the whole list of values and what they mean. You'll see that a "`39`" means that the person graduated high school; a "`43`" means that they have a Bachelor's degree. Without that "`Values`" information you'd have no way to know that. It also means that you must do a bit of work re-coding variables before you work with the data. The variable "TEAGE" (which is the person's age) has numbers like 35, 48, 19 – just what you'd expect. These values have a natural interpretation; you don't need a codebook for this one! The variable "TESEX" tells whether the person is male or female – but it doesn't use text, it just lists either the number 1 or 2. We could guess that one of those is male and the other female, but we'd have to go back to "Variable View" to look at "Values" for "TESEX" to find that a 1 indicates a male and a 2 indicates female.

Start with "TESEX" to create, instead, a dummy variable (that takes a value of just zero or one) called "female" that is equal to one if the person is female and zero if not. To do this, click "Transform" then "Compute…" which will bring up a dialog box. The "Target Variable" is the new variable you are creating; for this case, type in "female". The "Numeric Expression" allows considerable freedom in transforming variables. For this case, we will only need a logical expression: "TESEX = 2". You can either type in the variable name, "TESEX", or find the variable name in the list on the left of the dialog box and click the arrow to insert the name.

Later you might encounter cases where you want more complicated dummy variables and want to use logical relations "and" "or" "not" (the symbols "&", "|", "~") or the ">=" or multiplication or division. But in this case, we just need "TESEX = 2" which SPSS interprets as telling it to set a value of 1 in each case where that logical expression is true, and a value of zero in each case where that expression is false. If you go to "Data View" and scroll over (new variables are all the way on the right) you can check that it looks right.

Next we'll create the racial variables. We'll create dummy variables for "white", "African-American", "American Indian/Inuit/Hawaiian/Pacific Islander", and "Asian." We'll lump together the people who give multiple identities with those who give a single one (this is standard in much empirical work, although it is evolving rapidly).

So "Tranform/Compute…" and label "Target Variable" as "white" with "Numeric Expression" "PTDTRACE=1". Then "afam" is "(PTDTRACE=2) | (PTDTRACE=6) | (PTDTRACE=10) | (PTDTRACE=11) | (PTDTRACE=12) | (PTDTRACE=15) | (PTDTRACE=16) | (PTDTRACE=19)" – note the parentheses and the "or" symbol. "Asian" is "(PTDTRACE=4) | (PTDTRACE=8)". "Amindian" is "(PTDTRACE=3) | (PTDTRACE=5) | (PTDTRACE=7) | (PTDTRACE=9) | (PTDTRACE=13) | (PTDTRACE=14) | (PTDTRACE=17) | (PTDTRACE=18) | (PTDTRACE=20) | (PTDTRACE=21)". Many of these codings of multiple races could be argued – you can make changes if you wish. One reason to learn to do this yourself is to find out where minor changes could make a difference in the conclusions. Do you think that, say, average wages are different for these racial categories?



We create a dummy variable for "Hispanic". Again use "Transform/Compute..." and label "Target Variable" as "Hispanic" with "Numeric Expression" of "(PEHSPNON = 1)".

Earlier I mentioned that we can't find non-Hispanic whites by taking the total number of "white only" and subtracting "Hispanic" but how can we find the actual number of non-Hispanic whites? On the drop-down menu of SPSS find "Transform" then "Compute Variable" then in the dialog box, give the new variable (it calls it the "Target Variable") a name (e.g. "nonHispWhite") and a Numeric Expression, for example here " (RAC1P = 1) & (HISP = 1) ". The first expression evaluates, for each case, whether the variable, "RAC1P" which is the variable coding race, has a value of 1 (which corresponds to the label "White alone"). If it equals 1 then the expression is True, which is coded as 1; if RAC1P does not equal 1 then the expression is False, coded as zero. The second expression evaluates if "HISP" equals 1 or not. The "&" sign in the middle evaluates if both expressions are true or not. When we run this classification, we find that 38.9% are non-Hispanic white, a big difference from the previous 24%!

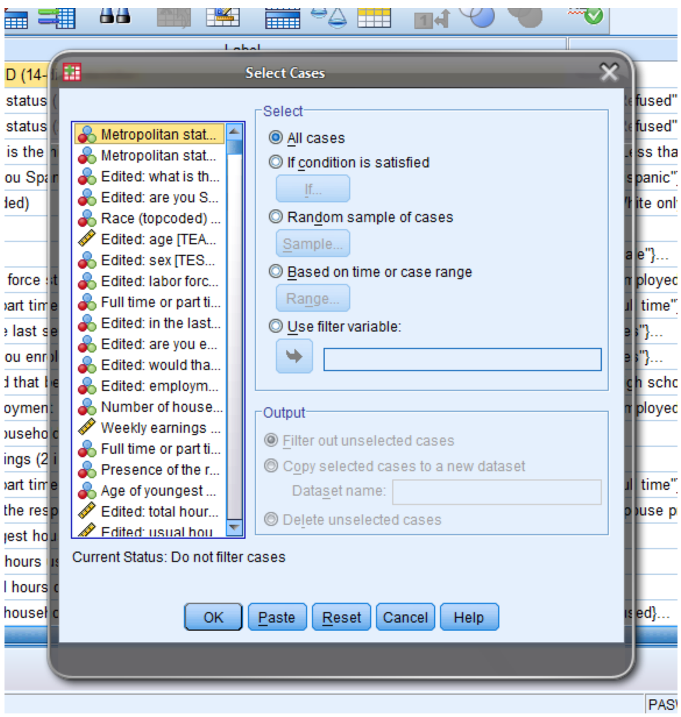
Create dummy variables for education: a dummy for no high school "ed_nohs", for high school but no further "ed_hs", for some college "ed_scol", for a bachelor's degree "ed_coll", and for more than a 4-year degree "ed_adv". "Transform/Compute...", set "Target Variable" as "ed_nohs" and "Numeric Expression" as " PEEDUCA <39". Then "ed_hs" is " PEEDUCA =39"; "ed_scol" is "(PEEDUCA >39)&(PEEDUCA <43)"; "ed_coll" is " PEEDUCA =43"; "ed_adv" is " PEEDUCA >43". Sometimes we distinguish various sorts of "some college" between people who got an Associate's degree versus those who took classes but never got any degree.

Then run "Descriptive Statistics" to make sure everything looks right – your dummy variables should have min=0 and max=1, for example!

Data Sub-Sets

Often we want to compare groups of people within the dataset to each other, for example looking at whether men or women get paid more or commute different or whatever. Comparisons are often more useful than just raw numbers because comparisons allow us to begin to judge which differences are substantial.

Do this with "`Data`" then "`Select Cases...`" to get a screen like this:



Usually we select cases "`If condition is satisfied`" so choose that, then click on "`If...`"

This brings up a dialog box that looks like the "Compute Variable" box from above. If we have already created a dummy variable that has values of only zeroes and ones then you can just put that into the "Select Cases" box. If you want a more complicated set then you can build it up using the logical notation that we discussed above. So suppose you want to look at just the subgroup of women between the ages of 18-35. Then we would enter "`(TESEX = 2) & (TEAGE > 18) & (TEAGE <= 35)`". Click "Continue". Make sure the output is "Filter out unselected cases" (you don't usually want to permanently delete the unselected cases!). Then all of your subsequent analyses will be done for just that subgroup.

Often an analysis will be more concerned with whether a particular item is done rather than how long – for example, when looking at working, whether a person has a second job (so time spent working second job is greater than zero) is probably more important than just how long they spent working at this second job. So often the "`if...`" statement will be of the form, "`X > 0`" for whatever variable, X, you're considering.

Later on, we will learn some more sophisticated ways of doing it but for now this is straightforward and clear. It will allow you to do the homework assignment.

Example

I will do an example to make this a bit clearer. We will look at the difference in how much time male and female college students spend watching TV. (I hope that for you the answer, how much time is wasted on TV, is "zero"!)

Open the ATUS 2003-2009 dataset.

First use "`Transform \ Compute ...`" to create a new variable, tv_time, which we set equal to the sum of T120303, watching non-religious TV, and T120304, watching religious TV. (Should we include T120308, playing computer games?)

Use "`Transform \ Compute ...`" to create another variable, educ_time, which is the sum of time spent doing things relevant to education, T060101 + T060102 + T060103 + T060104 + T060199 + T060301 + T060302 + T060303 + T060399. (Time spent in class and time spent doing homework, mainly.)

I'll also create "ratio_TV_study" that is the ratio of TV_time to educ_time.

Run "`Analyze \ Descriptive Statistics \ Descriptives ...`" to check that these seem sensible:

Descriptive Statistics

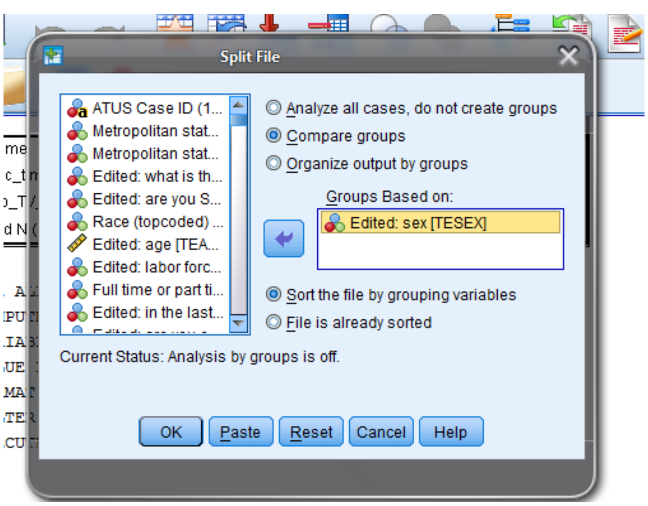
	N	Minimum	Maximum	Mean	Std. Deviation
tv_time	98778	.00	1417.00	165.2058	168.33963
educ_time	98778	.00	1090.00	16.3008	79.47292
ratio_TV_study	5974	.00	120.00	1.0450	3.00829
Valid N (listwise)	5974				

Note that the average for "educ_time" is low because most non-students will report zero time spent studying. All of those zero values returned errors when computing the ratio, so this has only 5974 reports of people with more than zero time studying.

Use "Data \ Select Cases ... " to select only college students (those for whom the 13th variable, TESCHLVL, is equal to 2).

Now to compare men and women I will use "`Data \ Split File ...`" to split into two groups and compare them – the program will do this automatically for all subsequent analysis.

This Split File screen is:



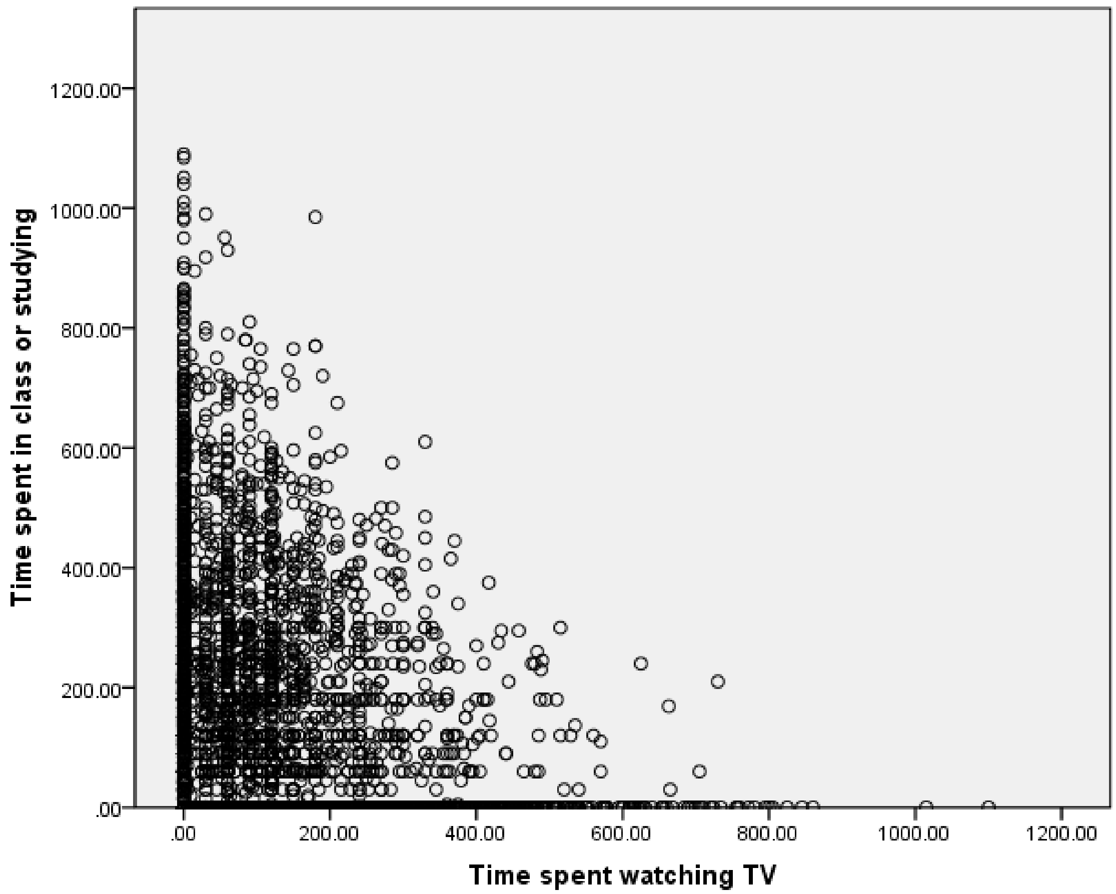
Now when I run the same "Descriptives" as before, this time I get the output subdivided:

Descriptive Statistics

Edited: sex		N	Minimum	Maximum	Mean	Std. Deviation
= "Male"	tv_time	2018	.00	860.00	127.1665	138.93259
	educ_time	2018	.00	1051.00	112.6056	186.01012
	ratio_TV_study	784	.00	75.00	.8390	3.02939
	Valid N (listwise)	784				
= "Female"	tv_time	3581	.00	1100.00	111.4739	124.86338
	educ_time	3581	.00	1090.00	104.8176	173.84758
	ratio_TV_study	1450	.00	120.00	.9117	4.04470
	Valid N (listwise)	1450				

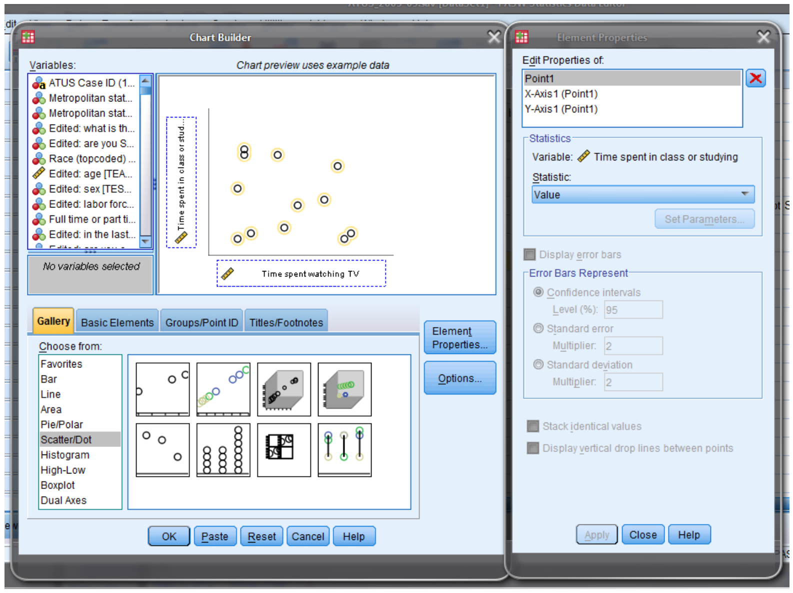
This shows that male college students watch an average of 127 minutes of TV per day and devote an average of 113 minutes to school; females watch 111 minutes of TV and devote 105 minutes to their studies. Men watch more TV but also spend a bit more time on school so the average ratio of time spent watching TV to time spent on school is .91 for women and .84 for men.

Finally I'll show a graph,



Note that there are quite a number of respondents who spent zero time studying or zero time watching TV. We would expect a downward relation since it is like a budget set: the more time is spent watching TV, the less is available to do anything else.

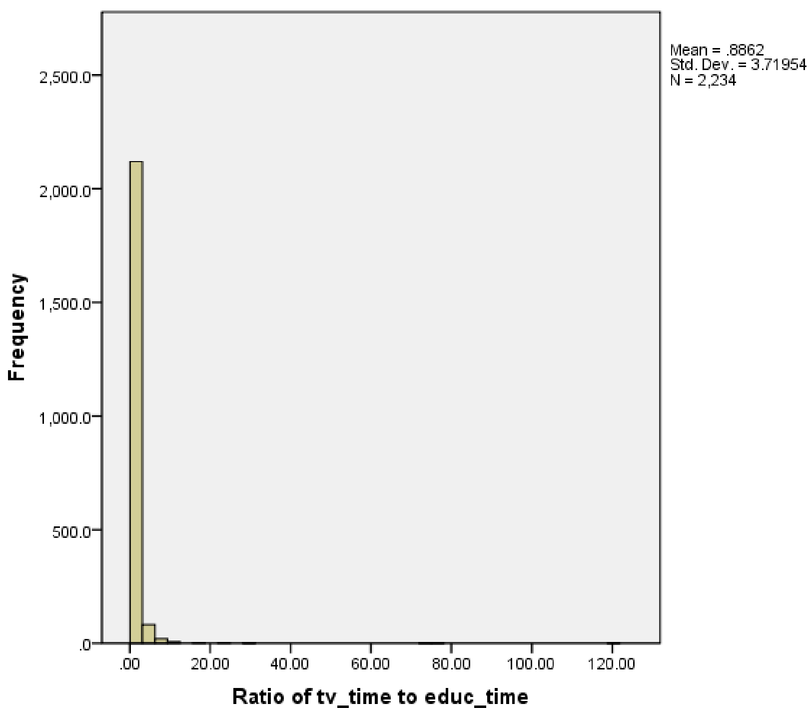
To get this graph, choose "`Graphs \ Chart Builder ...`" and drag the elements to where you want them, like this,



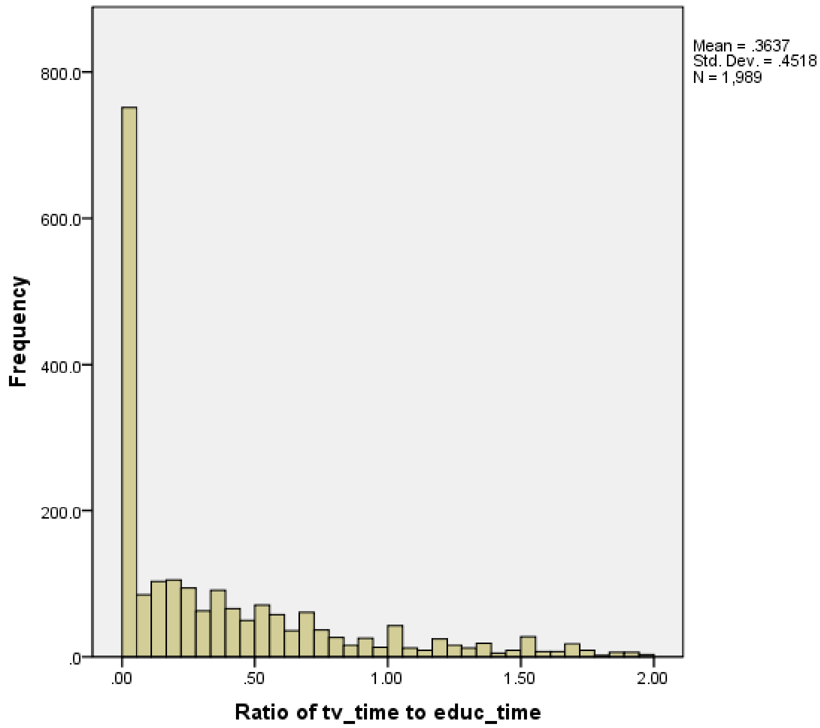
This is the first type of "Scatter/Dot" graph.

For this graph I removed the split, since it didn't look like there were significant differences between men and women in that regard – the same "`Data \ Split File . . .`" but now "Analyze all cases."

I can create a histogram of the ratio of time spent watching TV to time spent studying,



But this isn't much use since it's dominated by the few extreme values of people who spent 100 or more times as many minutes in TV as studying. So this histogram,



plots only those with a ratio less than 2.

(To make this chart, I used "`Graphs \ Chart Builder ...`" and then chose "Bar." When you put in just one variable on the x-axis it assumes you want a Histogram.)

Now you can go on to do your own analysis, maybe by race/ethnicity? Or go back and add in video game playing? Of the people who didn't watch TV, were there a larger fraction of men or women?

Some Shortcuts

You can use "`Analyze \ Descriptive Statistics \ Explore...`" which asks you to put in the "`Dependent List`" which are the variables, whose means you want to find, and then the "`Factor List`" which defines categories, by which the subgroup means are found. So, for example, if you wanted to look at the time sleeping, depending on whether there are kids in the house, you could put "`Time Sleeping`" into the "`Dependent List`" and then "`Presence of Household Children`" into the "`Factor List`".

You can get fancier if you create your own factors – suppose you wanted to look at time sleeping for African-American, Hispanic, Asian, and whites at 5 levels of education each (without highschool diploma, with just diploma, with some college, with 4-year degree, with advanced degree) – for a total of 4 x 5 = 20 different categories. So create a new variable that takes the values 1 through 20 and carefully code it up for each of those categories. Then put that into "Factor" in "Explore" and let the machine do your work.

SPSS also has "Analyze \ Compare Means" but we won't get to that yet (although you're welcome to explore it on your own!).

Other Datasets

The class will use a number of other data sets, which I have provided to you already formatted for SPSS. These are usually assembled by government bureaucrats who love their acronyms so they include names like Fed SCF, NHIS, BRFSS, NHANES, WVS, historical PUMS.

Overview of ATUS data

We will also use data from the "American Time Use Survey," or ATUS. This asks respondents to carefully list how they spent each hour of their time during the day; it's a tremendous resource. The survey data is collected by the US Bureau of Labor Statistics (BLS), a US government agency. You can find more information about it here, http://www.bls.gov/tus/.

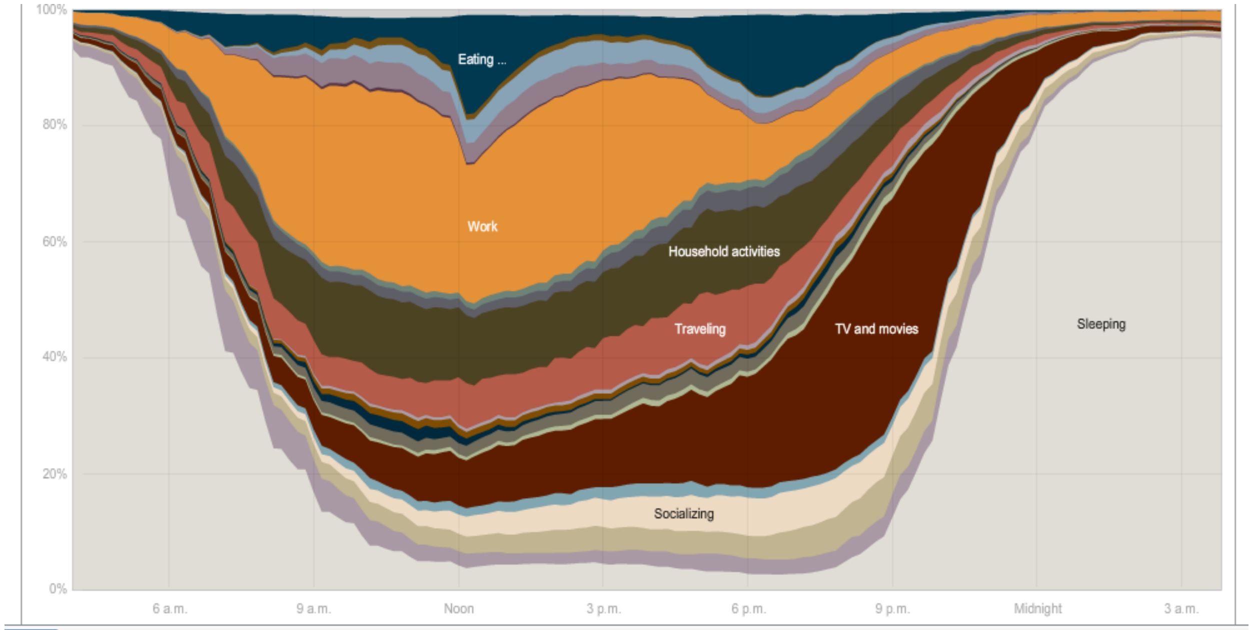
The dataset has information on 112,038 people interviewed from 2003-2010. This gives you a **ton** of information – we really need to work to get even the simplest information from it.

The dataset is ready to use in SPSS. Download it from the class page onto your computer. If it is zipped, then unzip it. Remember that if you're in the computer lab, just double-clicking on the SPSS file may not automatically start up SPSS; you'll get some error code. So use the Start bar to find SPSS and start it that way. Then open up your dataset once the program has loaded.

The ATUS has data telling how many minutes each person spent on various activities during the day. These are created from detailed logbooks that each person kept, recording their activities throughout the day.

They recorded how much time was spent with family members, with spouse, sleeping, watching TV, doing household chores, working, commuting, going to church/religious ceremonies, volunteering – there are hundreds of specific data items!

The NY Times had this graphic showing the different uses of time during the day [here http://www.nytimes.com/interactive/2009/07/31/business/20080801-metrics-graphic.html is the full interactive chart where you can compare the time use patterns of men and women, employed and unemployed, and other groups – a great way to lose an evening! The article is here http://www.nytimes.com/2009/08/02/business/02metrics.html?_r=2]



To use the data effectively, it is helpful to understand the ATUS classification system, where additional numbers at the right indicated additional specificity. The first two digits give generic broad categories. The general classification **T05** refers to time spent doing things related to work. **T0501** is specific to actual work; **T050101** is "Work, main job" then **T050102** is "Work, other job," **T050103** is "Security Procedures related to work," and **T050189** is "Working, Not Elsewhere Classified," abbreviated as n.e.c. (usually if the final digit is a nine then that means that it is a miscellaneous or catch-all category). Then there are activities that are strongly related to work, that a person might not do if they were not working at a particular job – like taking a client out to dinner or golfing. These get their own classification codes, **T050201**, **T050202**, **T050203**, **T050204**, or **T050289**. The list continues; there are "Income-generating hobbies, crafts, and food" and "Job interviewing" and "Job search activities." These have other classifications beginning with **T05** to indicate that they are work-related.

So for instance, to create a variable, "Time Spent Working" that we might label "T_work," you would add up T050101, T050102, T050103, T050189, T050201, T050202, T050203, T050204, T050289, T050301, T050302, T050303, T050304, T050389, T050403, T050404, T050405, T050481, T050499, and T059999. You might want to add in "Travel related to working" down in T180501. (No sane human would remember all these codings but you'd look at the "Labels" in SPSS and create a new variable.) It's tedious but not difficult in any way.

Some variables are even more detailed – playing sports is broken down into aerobics, baseball, basketball, biking, billiards, boating, bowling, ... all the way to wrestling, yoga, and

"Not Elsewhere Classified" for those with really obscure interests. Then there are similar breakdowns for watching those sports. Most people will have a zero value for most of these but they're important for a few people.

You can imagine that different researchers, exploring different questions, could want different aggregates. So the basic data has a very fine classification which you can add up however you want.

Fed SCF, Survey of Consumer Finances produced by the Federal Reserve

This survey is only made once every three years; the most recent data is from 2010. The survey gives a tremendous amount of information about people's finances: how much they have in bank accounts (and how many bank accounts), credit cards, mortgages, student loans, auto and other loans, retirement savings, mutual funds, other assets – the whole panoply of financial information. But there's a catch. As you probably know from class as well as from personal experience, wealth is very unequally distributed. Some people have few financial assets at all, not even a bank account. Many people have only a few basic financial instruments: a credit card, some basic loans and a simple bank account. Then a few wealthy people have tremendously complicated portfolios of assets.

How does a statistical survey deal with this? By unequal sampling then weighting – all of the samples I provide here do this to one degree or another, but it becomes very important in the Fed SCF. The idea is simple: from the perspective of a survey about finance, all people with no financial assets look the same – they have "zero" for most answers in the survey. So a single response is an accurate sample for lots and lots of people. But people with lots of financial assets have varied portfolios, so a single response is an accurate sample for only a small number of people. So if I were tasked with finding out about the financial system but could only survey 10 people, I might reasonably choose to sample 8 rich people with complicated portfolios and maybe 1 middle-class person and 1 poor person. I would keep in mind that the population of people in the country are not 80% rich, of course! In somewhat fancier statistics, I would weight each person, so the poor person would represent tens of millions of Americans, the middle-class person might represent more than a hundred million, and the rich people would each only represent a few million. If I wanted to extrapolate from the sample to the population, I would have to use these weights.

Many of the surveys we'll be using in class are weighted, and if you want to use them correctly you'll have to do the weighted versions. I'm skipping that for this class only because I think the cost outweighs the benefits for students early in their curriculum.

Actually using the Fed SCF survey can be difficult because the information is so richly detailed. You might want, say, a family's total debt, but instead get debt on credit card #1, card #2, all types of different loans, etc. so you have to add them up yourself. You have to do a bit of preliminary work.

NHIS National Health Interview Survey

This dataset has all sorts of medical and healthcare data – who has insurance, how often they're sick, doctor visits, pregnancy, weight/height. In the US many people have health insurance provided through their work so the economics of health and economics of insurance become tangled together.

BRFSS, Behavioral Risk Factor Surveillance System Survey

This dataset has many observations on a wide variety of risky behaviors: smoking, drinking, poor eating, flu shots, whether household has a 3-day supply of food and water… There is some economic data such as a person's income group.

NHANES – National Health And Nutrition Examination Survey

This has even more detail but on a smaller sample than the BRFSS. On whether people have healthy lifestyles: eat veg and fruit, their BMI, whether they smoke (various things), use drugs, sex (number of partners) – lots of things that are interesting enough to compensate for the dull (!?!?) stats necessary to analyze it.

There are other common data sources that are easily available online, which you can consider as you reflect upon your final project.

IPUMS

This is a tremendous data source, that has historical census data for past centuries, from http://www.ipums.org/. Some of the historical questions are weird (they asked if a person was "idiotic" or "dumb" – which sounds crazy but used to be scientific terms). It includes full names and addresses from long-ago census data.

WVS World Values Survey

This has a bit less economics but still lots of interesting survey data about attitudes of people of many issues; the respondents are global from scores of countries over several different years. There is some information about personal income, education and occupation so you can see how those correlate with, say, attitudes toward democracy, religiosity, or other hot issues.

Demographic and Health Surveys from USAID

These give careful data about people in developing countries, to look at, say, how economic growth impacts nourishment.

On Correlations: Finding Relationships between Two Variables

In a case where we have two variables, X and Y, we want to know how or if they are related, so we use covariance and correlation.

Suppose we have a simple case where X has a two-part distribution that depends on another variable, Y, where Y is what we call a "dummy" variable: it is either a one or a zero but cannot have any other value. (Dummy variables are often used to encode answers to simple "Yes/No" questions where a "Yes" is indicated with a value of one and a "No" corresponds with a zero. Dummy variables are sometimes called "binary" or "logical" variables.) X might have a different mean depending on the value of Y.

There are millions of examples of different means between two groups. GPA might be considered, with the mean depending on whether the student is a grad or undergrad. Or income might be the variable studied, which changes depending on whether a person has a college degree or not. You might object: but there are lots of other reasons why GPA or income could change, not just those two little reasons – of course! We're not ruling out any further complications; we're just working through one at a time.

In the PUMS data, X might be "wage and salary income in past 12 months" and Y would be male or female. Would you expect that the mean of X for men is greater or less than the mean of X for women?

Run this on SPSS …

In a case where X has two distinct distributions depending on whether the dummy variable, Y, is zero or one, we might find the sample average for each part, so calculate the average when Y is equal to one and the average when Y is zero, which we denote $\left(\bar{X}\,\middle|Y=0\right),\left(\bar{X}\,\middle|Y=1\right)$ *or* $\bar{X}_{Y=0},\bar{X}_{Y=1}$. These are called conditional means since they give the mean, conditional on some value.

In this case the value of $\bar{X}\,|Y=1$ is the same as the average of the two variables multiplied together, $X \cdot Y$.

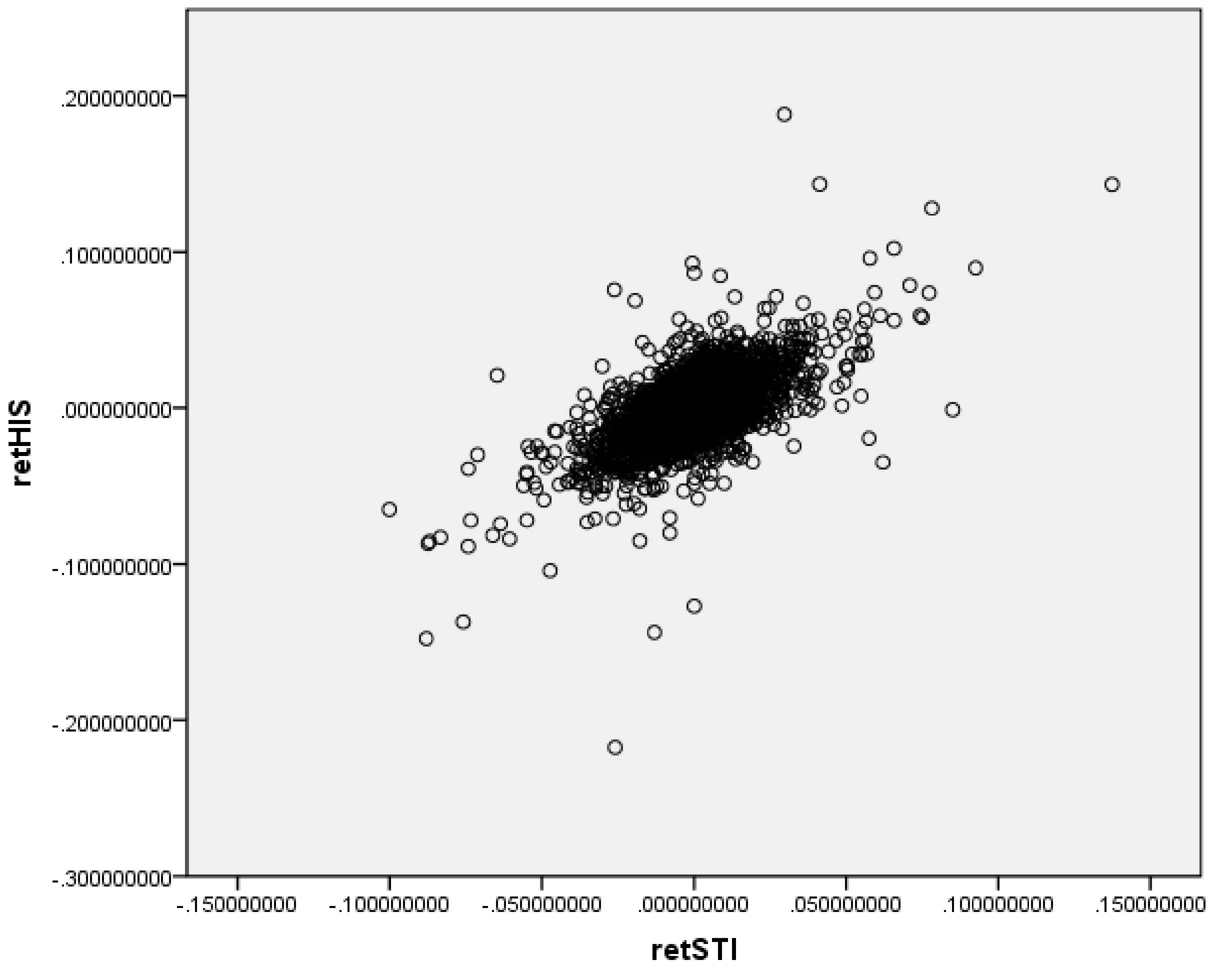
$$\overline{XY}=\frac{1}{N}\sum_{i=1}^{N}X_iY_i=\frac{1}{N}\sum_{i=1}^{N}X_i\left\{Y=1\right\}+\frac{1}{N}\sum_{i=1}^{N}X_i\left\{Y=0\right\}=\frac{1}{N}\sum_{i=1}^{N}X_i\left\{Y=1\right\}=\bar{X}_{Y=1}.$$

This is because the value of anything times zero is itself zero, so the term $\sum_{i=1}^{n}X_i\left\{Y=0\right\}$ drops out. While it is easy to see how this additional information is valuable when Y is a dummy variable, it is a bit more difficult to see that it is valuable when Y is a continuous variable – why might we want to look at the multiplied value, $X \cdot Y$?

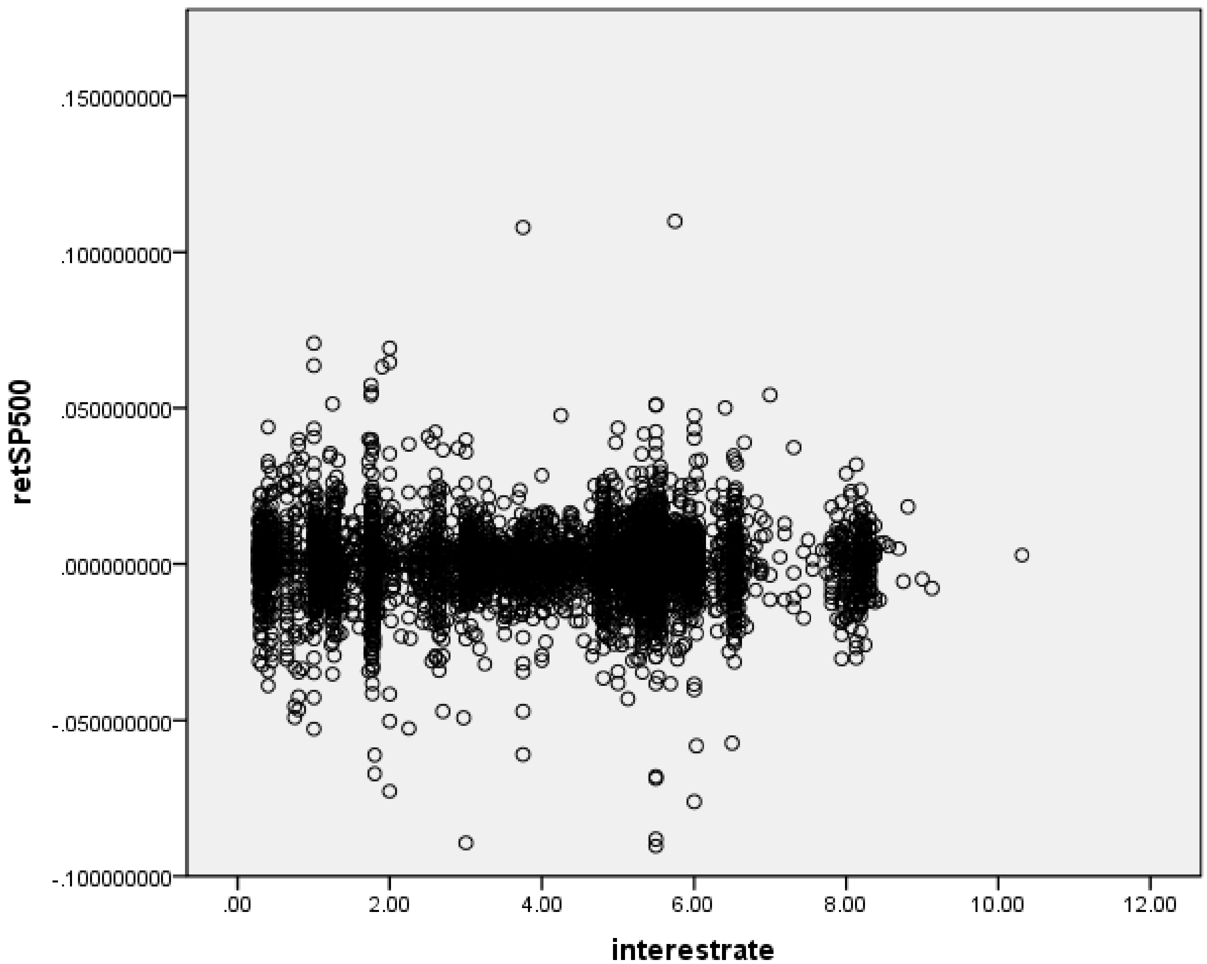
Use Your Eyes

We are accustomed to looking at graphs that show values of two variables and trying to discern patterns. Consider these two graphs of financial variables.

This plots the returns of Hong Kong's Hang Seng index against the returns of Singapore's Straits Times index (over the period from Dec 29, 1989 to Sept 1, 2010)



This next graph shows the S&P 500 returns and interest rates (1-month Eurodollar) during Jan 2, 1990 – Sept 1, 2010.



You don't have to be a highly-skilled econometrician to see the difference in the relationships. It would seem reasonable to state that the Hong Kong and Singapore stock indexes are closely linked; while US stock returns are not closely related to US interest rates.

We want to ask, how could we measure these relationships? Since these two graphs are rather extreme cases, how can we distinguish cases in the middle? And then there is one farther, even more important question: how can we try to guard against seeing relationships where, in fact, none actually exist? The second question is the big one, which most of this course (and much of the discipline of statistics) tries to answer. But start with the first question.

How can we measure the relationship?

Correlation measures how/if two variables move together.

Recall from above that we looked at the average of $X \cdot Y$ when Y was a dummy variable taking only the values of zero or one. Return to the case where Y is not a dummy but is a continuous variable just like X. It is still useful to find the average of $X \cdot Y$ even in the case where Y is from a continuous distribution and can take any value, $\overline{XY} = \frac{1}{n}\sum_{i=1}^{n} X_i Y_i$. It is a bit more useful if we re-write X and Y as differences from their means, so finding:

$$\frac{1}{N}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right).$$

This is the covariance, which is denoted cov(X,Y) or σ_{XY}.

A positive covariance shows that when X is above its mean, Y tends to also be above its mean (and vice versa) so either a positive number times a positive number gives a positive or a negative times a negative gives a positive.

A negative covariance shows that when X is above its mean, Y tends to be below its mean (and vice versa). So when one is positive the other is negative, which gives a negative value when multiplied.

A bit of math (extra):

$$\frac{1}{N}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right) =$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(X_iY_i - \bar{X}Y_i - X_i\bar{Y} + \bar{X}\bar{Y}\right) =$$

$$\frac{1}{N}\sum_{i=1}^{N} X_iY_i - \frac{1}{N}\sum_{i=1}^{N} \bar{X}Y_i - \frac{1}{N}\sum_{i=1}^{N} X_i\bar{Y} + \frac{1}{N}\sum_{i=1}^{N} \bar{X}\bar{Y} =$$

$$\frac{1}{N}\sum_{i=1}^{N} X_iY_i - \bar{X}\frac{1}{N}\sum_{i=1}^{N} Y_i - \bar{Y}\frac{1}{N}\sum_{i=1}^{N} X_i + \bar{X}\bar{Y}\frac{1}{N}\sum_{i=1}^{N} 1 =$$

$$\frac{1}{N}\sum_{i=1}^{N} X_iY_i - \bar{X}\bar{Y} - \bar{Y}\bar{X} + \bar{X}\bar{Y} =$$

$$\frac{1}{N}\sum_{i=1}^{N} X_iY_i - \bar{X}\bar{Y}$$

(a strange case because it makes FOIL look like just FL!)

Covariance is sometimes scaled by the standard deviations of X and Y in order to eliminate problems of measurement units, so the correlation is:

$$\frac{\frac{1}{N}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sigma_X \sigma_Y} = \frac{\sigma_{XY}}{\sigma_X \sigma_y} = \rho_{XY}$$ or Corr(X,Y),

where the Greek letter "rho" denotes the correlation coefficient. With some algebra you can show that ρ is always between negative one and positive one; $-1 \le \rho_{XY} \le 1$.

Two variables will have a perfect correlation if they are identical; they would be perfectly inversely correlated if one is just the negative of the other (assets and liabilities, for example). Variables with a correlation close to one (in absolute value) are very similar; variables with a low or zero correlation are nearly or completely unrelated.

Sample covariances and sample correlations

Just as with the average and standard deviation, we can estimate the covariance and correlation between any two variables. And just as with the sample average, the sample covariance and sample correlation will have distributions around their true value.

Go back to the case of the Hang Seng/Straits Times stock indexes. We can't just say that when one is big, the other is too. We want to be a bit more precise and say that when one is above its mean, the other tends to be above its mean, too. We might additionally state that, when the standardized value of one is high, the other standardized value is also high. (Recall that the standardized value of one variable, X, is $Z_{X,i} = \frac{X_i - \bar{X}}{s_X}$, and the standardized value of Y is $Z_{Y,i} = \frac{Y_i - \bar{Y}}{s_Y}$.)

Multiplying the two values together, $Z_{X,i} Z_{Y,i}$, gives a useful indicator since if both values are positive then the multiplication will be positive; if both are negative then the multiplication will again be positive. So if the values of Z_X and Z_Y are perfectly linked together then multiplying them together will get a positive number. On the other hand, if Z_X and Z_Y are oppositely related, so whenever one is positive the other is negative, then multiplying them together will get a negative number. And if Z_X and Z_Y are just random and not related to each other, then multiplying them will sometimes give a positive and sometimes a negative number.

Sum up these multiplied values and get the (population) correlation, $\frac{1}{N}\sum_{i=1}^{N} Z_{X,i} Z_{Y,i}$.

This can be written as

$$\frac{1}{N}\sum_{i=1}^{N} Z_{X,i} Z_{Y,i} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{X_i - \bar{X}}{s_x}\right)\left(\frac{Y_i - \bar{Y}}{s_Y}\right) = \frac{1}{N}\frac{1}{s_x s_Y}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right).$$

The population correlation between X and Y is denoted ρ_{XY}; the sample correlation is r_{XY}. Again the difference is whether you divide by N or (N – 1). Both correlations are always between -1 and +1; $-1 \le \rho \le 1; -1 \le r \le 1$.

We often think of drawing lines to summarize relationships; the correlation tells us something about how well a line would fit the data. A correlation with an absolute value near 1 or -1 tells us that a line (with either positive or negative slope) would fit well; a correlation near zero tells us that there is "zero relationship."

The fact that a negative value can infer a relationship might seem surprising but consider for example poker. Suppose you have figured out that an opponent makes a particular gesture when her cards are no good – you can exploit that knowledge, even if it is a negative relationship. In finance, if a fund manager finds two assets that have a strong negative correlation, that one has high returns when the other has low returns, then again this information can exploited by taking offsetting positions.

You might commonly see a "covariance matrix" if you were working with many variables; the matrix shows the covariance (or correlation) between each pair. So if you have 4 variables, named (unimaginatively) X1, X2, X3, and X4, then the covariance matrix would be:

	X1	X2	X3	X4
X1	$\sigma 11$			
X2	$\sigma 21$	$\sigma 22$		
X3	$\sigma 31$	$\sigma 32$	$\sigma 33$	
X4	$\sigma 41$	$\sigma 42$	$\sigma 34$	$\sigma 44$

Where the matrix is "lower triangular" because cov(X,Y)=cov(Y,X) [return to the formulas if you're not convinced] so we know that the upper entries would be equal to their symmetric lower-triangular entry (so the upper triangle is left blank since the entries would be redundant). And we can also show [again, a bit of math to try on your own] that cov(X,X) = var(X) so the entries on the main diagonal are the variances.

If we have a lot of variables (15 or 20) then the covariance matrix can be an important way to easily show which ones are tightly related and which ones are not.

As a practical matter, sometimes perfect (or very high) correlations can be understood simply by definition: a survey asking "Do you live in a city?" and "Do you live in the countryside?" will get a very high negative correlation between those two questions. A firm's Assets and Liabilities ought to be highly correlated. But other correlations can be caused by the nature of the sampling.

Higher Moments

The third moment is usually measured by skewness, which is a common characteristic of financial returns: there are lots of small positive values balanced by fewer but larger negative values. Two portfolios could have the same average return and same standard deviation, but if one is not symmetric distribution (so has a non-zero skewness) then it would be important to understand this risk.

The fourth moment is kurtosis, which measures how fat the tails are, or how fast the probabilities of extreme values die off. Again a risk manager, for example, would be interested in understanding the differences between a distribution with low kurtosis (so lots of small changes) versus a distribution with high kurtosis (a few big changes).

If these measures are not perfectly clear to you, don't get frustrated – it is difficult, but it is also very rewarding. As the Financial Crisis has shown, many top risk managers at name-brand institutions did not understand the statistical distributions of the risks that they were taking on. They plunged the global economy into recession and chaos because of it.

These are called "moments" to reflect the origin of the average as being like weights on a lever or "moment arm". The average is the first moment, the variance is the second, skewness is third, kurtosis is fourth, etc. If you take a class using Calculus to go through Probability and Statistics, you will learn moment-generating functions.

More examples of correlation:

It is common in finance to want to know the correlation between returns on different assets.

First remember the difference between the returns and the level of an asset or index!

An investment in multiple assets, with the same return but that are uncorrelated, will have the same return but with less overall risk. We can show this on Excel; first we'll do random numbers to show the basic idea and then use specific stocks.

How can we create normally-distributed random numbers in Excel? RAND() gives random numbers between zero and one; NORMSINV(RAND()) gives normally distributed random numbers. (If you want variables with other distributions, use the inverse of those distribution functions.) Suppose that two variables each have returns given as 2% + a normally-distributed random number; this is shown in Excel sheet, lecturenotes3.xls

With finance data, we use the return not just the price. This is because we assume that investors care about returns per dollar not the level of the stock price.

Important Questions

- When we calculate a correlation, what number is "big"? Will see random errors – what amount of evidence can convince us that there is really a correlation?
- When we calculate conditional means, and find differences between groups, what difference is "big"? What amount of evidence would convince us of a difference?

Example:

Mazar, Amir, Ariely (2005) "Dishonesty of Honest People" [SSRN-id979648.pdf, available online]

Students solve math problems and report how many, of 20, were solved (offered a small reward for success). Here is a sample question: **Which 2 numbers add to 10**? You can see that finding the answer is tedious but doesn't require advanced mathematical knowledge.

1.69	1.82	2.91
4.67	4.81	3.05
5.82	5.06	4.28
6.36	5.19	4.57

In one setup, the students first threw out the answer sheet and then just said how many they'd solved; in the other setup they handed over the sheet to be checked – so it was easier to cheat in the first case. Students who had to hand in the sheet reported solving an average of 3.1 out of 20 problems in the short time given; students who threw out the sheet reported 4.2.

Are people more dishonest, when given a chance to be? Really? What information do we need, to be more confident about our knowledge? Ariely did another study looking at whether wearing counterfeit sunglasses made people more likely to cheat.

To answer these, we need to think about randomness – in other perceptual problems, what would be called noise or blur.

Learning Outcomes (from CFA exam Study Session 2, Quantitative Methods)

Students will be able to:

- calculate and interpret relative frequencies, given a frequency distribution, and describe the properties of a dataset presented as a histogram;
- define, calculate, and interpret measures of central tendency, including the population mean, sample mean, median, and mode;
- define, calculate, and interpret measures of variation, including the population standard deviation and the sample standard deviation;
- define and interpret the covariance and correlation;
- define a random variable, an outcome, an event, mutually exclusive events, and exhaustive events;
- distinguish between dependent and independent events;

Probability

Beyond presenting some basic measures such as averages and standard deviations, we want to try to understand how much these measures can tell us about the larger world. How likely is it, that we're being fooled, into thinking that there's a relationship when actually none exists? To think through these questions we must consider the logical implications of randomness and often use some basic statistical distributions (discrete or continuous).

Think Like a Statistician

The basic question that a Statistician must ask is "How likely is it, that I'm being fooled?" Once we accept that the world is random (rather than a manifestation of some god's will), we must decide how to make our decisions, knowing that we cannot guarantee that we will always be right. There is some risk that the world will seem to be one way, when actually it is not. The stars are strewn randomly across the sky but some bright ones seem to line up into patterns. So too any data might sometimes line up into patterns.

Statisticians tend to stand on their heads and ask, suppose there were actually no relationship? (Sometimes they ask, "suppose the conventional wisdom were true?") This statement, about "no relationship," is called the **Null Hypothesis**, sometimes abbreviated as H_0. The Null Hypothesis is tested against an **Alternative Hypothesis**, H_A.

Before we even begin looking at the data we can set down some rules for this test. We know that there is some probability that nature will fool me, that it will seem as though there is a relationship when actually there is none. The statistical test will create a model of a world where there is actually no relationship and then ask how likely it is that we could see what we actually see, "How likely is it, that I'm being fooled?" What if there were actually no relationship, is there some chance that I could see what I actually see?

Randomness in Sports

As an example, consider sports events. As any sports fan knows, a team or individual can get lucky or unlucky. The baseball World Series, for example, has seven games. It is designed to ensure that, by the end, one team or the other wins. But will the better team always win?

First make a note about subjectivity: if I am a fan of the team that won, then I will be convinced that the better team won; if I'm a fan of the losing team then I'll be certain that the better team got unlucky. But fans of each team might agree, if they discussed the question before the Series were played, that luck has a role.

Will the better team win? Clearly a seven-game Series means that one team or the other will win, even if they are exactly matched (if each had precisely a 50% chance of winning). If two representatives tossed a coin in the air seven times, then one or the other would win at least four tosses – maybe even more. We can use a computer to simulate seven coin-tosses by having it pick a random number between zero and one and defining a "win" as when the random number is greater than 0.5.

Or instead of having a computer do it, we could use a bit of statistical theory.

Some math

Suppose we start with just one coin-toss or game (baseball uses 7 games to decide a champion; football uses just one). Choose to focus on one team so that we can talk about "win" and "loss". If this team has a probability of winning that is equal to p, then it has a probability of losing equal to $(1-p)$. So even if p, the probability of winning, is equal to 0.6, there is still a 40% chance that it could lose a single game. In fact unless the probability of winning is 100%, there is some chance, however remote, that the lesser team will win.

What about if they played two games? What are the outcomes? The probability of a team winning both games is $p*p = p^2$. If the probability were 0.5 then the probability of winning twice in a row would be 0.25.

A table can show this:

	Win Game 1 {p}	Lose Game 1 {1-p}
Win Game 2 {p}	outcome: W,W	L,W
Lose Game 2 {1-p}	W,L	L,L

This is a fundamental fact about how probabilities are represented mathematically: if the probabilities are not related (i.e. if the tossed coin has no memory) then the probability of both events happening is found my multiplying the probabilities of each individual outcome. (What if they're not unrelated, you may ask? What if the first team that wins gets a psychological boost in the next so they're more likely to win the second game? Then the math gets more complicated – we'll come back to that question!)

The math notation for two events, call them A and B, both happening is:

$$\Pr\{A \text{ and } B\} = \Pr\{A \cap B\}$$

The fundamental fact of independence is then represented as:

$$\Pr\{A \cap B\} = \Pr\{A\}\Pr\{B\} \quad \text{if } A \text{ and } B \text{ are independent}$$

where we use the term "independent" for when there is no relationship between them.

The probability that a team could lose both games is (1-p)*(1-p) = $(1-p)^2$. The probability that the teams could split the series (each wins just one) is p*(1-p) + (1-p)*p = 2p(1-p). There are two ways that each team could win just one game: either the series splits (Win,Loss) or (Loss,Win).

For three games the outcomes become more complicated: now there are 8 combinations of win and loss:

(W,W,W)	(W,W,L)	(W,L,W)	(L,W,W)	(W,L,L)	(L,W,L)	
p*p*p	p*p*(1-p)	p*(1-p)p	(1-p)p*p	p(1-p)(1-p)	(1-p)p(1-p)	

and the probabilities are in the row below.

The team will win the series in any of the left-most 4 outcomes so its overall probability of winning the series is

$$p^3 + 3p^2(1-p)$$

while its probability of losing the series is

$$3p(1-p)^2 + (1-p)^3.$$

Clearly if p is 0.5 so that p=(1-p) then the chances of either team winning the three-game series are equal. If the probabilities are not equal then the chances are different, but as long as there is a probability not equal to one or zero (i.e. no certainty) then there is a chance that the worse team could win.

If you keep on working out the probabilities for longer and longer series you might notice that the coefficients and functional forms are right out of Pascal's Triangle. This is your first notice of just how "normal" the Normal Distribution is, in the sense that it jumps into all sorts of places where you might not expect it. The terms of Pascal's Triangle begin (as N becomes large) to have a normal distribution! We'll come back to this again…

Terms and Definitions

Some basics: a sample space is the entire list of possible outcomes (can be whole long list or even mathematical sets such as real numbers); events are subsets of the sample space. Simple event is a single outcome (one dice comes up 6); a compound event is several outcomes (both dice come up 6). Notate an event as A. The complement of the event is the set of all events that are not in A; this is A'.

The events must be **mutually exclusive and exhaustive**, so a good deal of the hard work in probability is just figuring out how to list all of the events.

Mutually exclusive means that the events must be clearly defined so that the data observed can be classified into just one event. Exhaustive means that every possible data observed must fit into some event. The "mutually exclusive" part means that probabilities can be added up, so that if the probability of rolling a "1" on a dice is 1/6 and the probability of rolling a 6 is 1/6, then the probability of rolling either a 1 or 6 is 2/6 = 1/3. The "exhaustive" part of defining the events means that the sum of all the events must equal one.

For example, suppose we roll two dice. We might want to think of "die #1 comes up as 6" as one event [in English, the singular of "dice" is "die" – how morbid gambling can be!]. But the other die can have 6 different values without changing the value of the first die. So a better list of events would be the integers from 2 to 12, the sum of the dice values – with the note that there are many ways of achieving some of the events (a 7 is a 6 &1 or a 5&2, or 4&3, or 3&4, or 2&5, or 1&6) while other events have only one path (each die comes up 6 to make 12).

A **sample space** is the set of all possible events. The sum of the probability of all of the events in the sample space is equal to one. There is a 100% chance that something happens (provided we've defined the sample space correctly). So if a lottery brags that there is a 2% chance that "you might be a winner!" this is equivalent to stating that there is a 98% chance that you'll lose.

Events have **probability**; this must lie between zero and one (inclusive); so $0 \le P \le 1$. The probability of all of the events in the sample space must sum to one. This means that the probability of an event and its complement must sum to one: $P\{A\} + P\{A'\} = 1$.

Probabilities come from empirical results (relative frequency approach) or the classical (a priori or postulated) assignment or from subjective beliefs that people have.

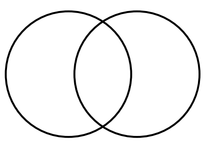
In empirical approach, the **Law of Large Numbers** is important: as the number of identical trials increases, the estimated frequency approaches its theoretical value. You can try flipping coins and seeing how many come up heads *(flip a bunch at a time to speed up the process)*; it should be 50%.

We are often interested in finding the probability of two events both happening; this is the "**intersection**" of two events; the logical "and" relationship; two things both occurring. In the PUMS data we might want to find how many females have a college degree; in poker we might care about the chance of an opponent having an ace as one of her hole cards and the dealer turning up a king. We notate the intersection of A and B as $A \cap B$ and want to find $P\{A \cap B\}$. In SPSS this is notated with "&".

The "**union**" of two events is the logical "or" so it is either of two events occurring; this is $A \cup B$ so we might consider $P\{A \cup B\}$ or, in SPSS, "|". In the PUMS data we might want to combine people who report themselves as having race "black" with those who report "black – white". In cards, it is the probability that any of my 3 opponents has a better hand.

Married people can buy life insurance policies that pay out either when the first person dies or after both die – logical *and* vs *or*.

Venn Diagrams (Ballantine)



General Law of Addition

$$P\{A \cup B\} = P\{A\} + P\{B\} - P\{A \cap B\}$$

and so $P\{A \cap B\} = P\{A\} + P\{B\} - P\{A \cup B\}$

Mutually Exclusive (Special Law of Addition),

If $A \cap B = \phi$ then $P\{A \cap B\} = 0$ and $P\{A \cup B\} = P\{A\} + P\{B\}$

Conditional Probability

$P\{A|B\} = \frac{P\{A \cap B\}}{P\{B\}}$ if $P\{B\} \neq 0$. See Venn Diagram.

Independent Events

A is independent of B if and only if $P\{A|B\} = P\{A\}$

If we have multiple random variables then we can consider their **Joint Distribution**: the probability associated with each outcome in both sample spaces. So a coin flip has a simple discrete distribution: a 50% chance of heads and a 50% chance of tails. Flipping 2 coins gives a joint distribution: a 25% chance of both coming up heads, a 25% chance of both coming up tails, and a 50% chance of getting one head and one tail.

The probability of multiple independent events is found by multiplying the probabilities of each event together. So the chance of rolling two 6 on two dice is $\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$. The probability of getting to the computer lab on the 6th floor of NAC from the first floor, without having to walk up a broken escalator, can be found this way too. Suppose the probability of an escalator not working is p; then the probability of it working is $(1-p)$ and the probability of five escalators each working is $(1-p)^5$. So even if the probability of a breakdown is small (5%), still the probability of having every escalator work is just $(1-5\%)^5=(95\%)^5=(0.95)^5=\left(\frac{95}{100}\right)^5=0.7738=77.38\%$ so this implies that you'd expect to walk more than once a week.

A simple representation of the joint distribution of two coin flips is a table:

	coin 1 Heads	coin 1 Tails
coin 2 Heads	H,H at 25%	H,T at 25%
coin 2 Tails	T,H at 25%	T,T at 25%

Where, since the outcomes are independent, we can just multiply the probabilities.

The Joint Distribution tells the probabilities of all of the different outcomes. A **Marginal Distribution** answers a slightly different question: given some value of one of the variables, what are the probabilities of the other variables?

When the variables are independent then the marginal distribution does not change from the joint distribution. Consider a simple example of X and Y discrete variables. X takes on values of 1 or 2 with probabilities of 0.6 and 0.4 respectively. Y takes on values of 1, 2, or 3 with probabilities of 0.5, 0.3, and 0.2 respectively. So we can give a table like this:

	X=1 (60%)	X=2 (40%)
Y=1 (50%)	(1,1) at probability 0.3	(2,1) at probability 0.2
Y=2 (30%)	(1,2) at probability 0.18	(2,2) at probability 0.12
Y=3 (20%)	(1,3) at probability 0.12	(2,3) at probability 0.08

On the assumption that X and Y are independent. The probabilities in each box are found by multiplying the probability of each independent event.

If instead we had the two variables, A and B, not being independent then we might have a table more like this:

	A=1	A=2
B=1	(1,1) at probability 0.25	(2,1) at probability 0.13
B=2	(1,2) at probability 0.23	(2,2) at probability 0.12
B=3	(1,3) at probability 0.17	(2,3) at probability 0.1

We will examine the differences.

If we add up the probabilities along either rows or columns then we get the **marginal probabilities** (which we write in the *margins*, appropriately enough). Then we'd get:

	X=1 (60%)	X=2 (40%)	
Y=1 (50%)	(1,1) at probability 0.3	(2,1) at probability 0.2	0.5
Y=2 (30%)	(1,2) at probability 0.18	(2,2) at probability 0.12	0.3
Y=3 (20%)	(1,3) at probability 0.12	(2,3) at probability 0.08	0.2
	0.6	0.4	

Which just re-states our assumption that the variables are independent – and shows that, where there is independence, the probability of either variable alone does not depend on the value that the other variable takes on. In other words, knowing X does not give me any information about the value that Y will take on, and vice versa.

If instead we do this for the A,B case we get:

	A=1	A=2	
B=1	(1,1) at probability 0.25	(2,1) at probability 0.13	0.38
B=2	(1,2) at probability 0.23	(2,2) at probability 0.12	0.35
B=3	(1,3) at probability 0.17	(2,3) at probability 0.1	0.27
	0.65	0.35	

Where we double check that we've done it right by seeing that the sum of either of the marginals is equal to one (65% + 35% = 100% and 38% + 35% + 27% = 100%).

So the marginal distributions sum the various ways that an outcome can happen. For example, we can get A=1 in any of 3 ways: either (1,1), (1,2) or (1,3). So we add the probabilities of each of these outcomes to find the total chance of getting A=1.

But if we want to understand how A and B are related, it might be more useful to consider this as a prediction problem: would knowing the value that A takes on help me guess the value of B? Would knowing the value that B takes on help me guess the value of A?

These are abstract questions but they have vitally important real-life analogs. In airport security, is the probability that someone is a terrorist independent of whether they are Muslim? Is the probability that someone is pulled out of line for a thorough search independent of whether they are Muslim? *(The TSA might have different beliefs than you or me!)* In medicine, is the probability that someone gets cancer independent of whether they eat lots of vegetables? In economics, is the probability that someone defaults on their mortgage independent of the mortgage originator (Fannie, Freddie, mortgage broker, bank)? Is the probability of the country pulling out of recession independent of whether the Fed raises rates? In poker, if my opponent just raised the bid, what is the probability that her cards are better than mine?

For these questions we want to find the conditional distribution: what is the probability of some outcome, given a particular value for some other random variable?

Just from the phrasing of the question, you should be able to see that if the two variables are independent then the conditional distribution should not change from the marginal distribution – as is the case of X and Y. Flipping a coin does not help me guess the outcome of a roll of the dice. (Cheering in front of a sports game on TV does not affect the outcome, for another example – although plenty of people act as though they don't believe that!)

How do we find the conditional distribution? Take the value of the joint distribution and divide it by the marginal distribution of the relevant variable.

For example, suppose we want to find the probability of B outcomes, conditional on A=1. Since we know that A=1, there is no longer a 65% probability of A -- it happened. So we divide each joint probability by 0.65 so that the sum will be equal to 1. So the probabilities are now:

	A=1	A=2	
B=1	(1,1) at probability 0.25/.65	(2,1) at probability 0.13	0.38
B=2	(1,2) at probability 0.23/.65	(2,2) at probability 0.12	0.35
B=3	(1,3) at probability 0.17/.65	(2,3) at probability 0.1	0.27
	0.65/.65	0.35	

so now we get the conditional distribution:

	A=1	A=2	
B=1	(1,1) @ 0.3846	(2,1) at probability 0.13	0.38
B=2	(1,2) @ 0.3538	(2,2) at probability 0.12	0.35
B=3	(1,3) @ 0.2615	(2,3) at probability 0.1	0.27
		0.35	

We could do the same to find the conditional distribution of B, given that A=2:

	A=1	A=2	

B=1	(1,1) at probability 0.25	(2,1) @ 0.13/.35 =.3714	0.38
B=2	(1,2) at probability 0.23	(2,2) @ 0.12/.35 = .3429	0.35
B=3	(1,3) at probability 0.17	(2,3) @ 0.1/.35 = .2857	0.27
	0.65		

These conditional probabilities are denoted as $\Pr\{B|A=2\}$ for example. We could find the expected value of B given that A equals 2, $E[B|A=2]$, just by multiplying the value of B by its probability of occurrence, so $E[B|A=2]=(1\cdot.3714)+(2\cdot.3429)+(3\cdot.2857)$.

We could find the conditional probabilities of A given B=1 or given B=2 or given B=3. In those cases we would sum across the rows rather than down the columns.

More pertinently, we can get crosstabs (on SPSS, "Analyze" then "Descriptive Statistics" then "Crosstabs") on two variables, for example the native/foreign born in each borough,

		foreign_born		
		0	1	Total
Boroughs	Bronx	33955	15928	49883
	Manhattan	40511	15632	56143
	Staten Is	16074	3971	20045
	Brooklyn	62464	37324	99788
	Queens	48193	41719	89912
Total		201197	114574	315771

To get the joint probabilities, we divide the counts by the grand total,

	native	foreign
Bronx	0.1075	0.0504
Manhattan	0.1283	0.0495
Staten Is	0.0509	0.0126
Brooklyn	0.1978	0.1182
Queens	0.1526	0.1321

Then get the marginals:

	native	foreign

Bronx	0.1075	0.0504	0.1580
Manhattan	0.1283	0.0495	0.1778
Staten Is	0.0509	0.0126	0.0635
Brooklyn	0.1978	0.1182	0.3160
Queens	0.1526	0.1321	0.2847
	0.6372	0.3628	

These show that, in NYC, 64% are natives and 36% are foreign-born. The most populous boroughs are Brooklyn and Queens, each with about 30% of the city's population, while Manhattan and the Bronx each have about 15% and tiny Staten Island has just over 6%.

Then the conditional probabilities. Conditional on being native born,

	native	foreign	
Bronx	0.1688	0.0504	0.1580
Manhattan	0.2013	0.0495	0.1778
Staten Is	0.0799	0.0126	0.0635
Brooklyn	0.3105	0.1182	0.3160
Queens	0.2395	0.1321	0.2847
	0.6372	0.3628	

So 31% of the natives live in Brooklyn, 24% in Queens, 20% in Manhattan, 17% in the Bronx, and 8% in Staten Island. So a larger fraction of natives (relative to overall population share) is in Manhattan and Staten Island while a much lower fraction of native-born are in Queens.

Conditional on being foreign born,

	native	foreign	
Bronx	0.1075	0.1390	0.1580
Manhattan	0.1283	0.1364	0.1778
Staten Is	0.0509	0.0347	0.0635
Brooklyn	0.1978	0.3258	0.3160
Queens	0.1526	0.3641	0.2847
	0.6372	0.3628	

So 36% of immigrants live in Queens (relative to 28% of the population overall), 33% in Brooklyn, 14% in the Bronx and Manhattan, and just 3% in Staten Island.

The relative fractions of native/immigrant by borough (so conditional probabilities) is

	native	foreign	
Bronx	0.6807	0.3193	0.1580
Manhattan	0.7216	0.2784	0.1778
Staten Is	0.8019	0.1981	0.0635

Brooklyn	0.6260	0.3740	0.3160
Queens	0.5360	0.4640	0.2847
	0.6372	0.3628	

So the borough with the highest fraction of immigrants is Queens (a 54-46 split), followed by Brooklyn, the Bronx, Manhattan, and Staten Island (where natives outnumber immigrants by 4-to-1).

Conditional probabilities can also be calculated with what is called **Bayes' Theorem**:

$$P\{B|A\} = \frac{P\{A|B\} \cdot P\{B\}}{P\{A\}}.$$

This can be understood by recalling the definition of conditional probability, $P\{A|B\} = \frac{P\{A \cap B\}}{P\{B\}}$, so $P\{B|A\} = \frac{P\{A \cap B\}}{P\{A\}}$, that the conditional probability equals the joint probability divided by the marginal probability.

The power of Bayes' Theorem can be understood by thinking about medical testing. Suppose a genetic test screens for some disease with 99% accuracy. Your test comes back positive – how worried should you be? The surprising answer is not 99% worried; in fact often you might be more than likely to be healthy! Suppose that the disease is rare so only 1 person in 1000 has it (so 0.1%). So out of 1000 people, one person has the disease and the test is 99% likely to identify that person. Out of the remaining 999 people, 1% will be misidentified as having the disease, so this is 9.99 – call it 10 people. So eleven people will test positive but only one will actually have the disease so the probability of having the disease given that the test comes up positive, $P\{sick|test+\}$, is $\frac{P\{test+|sick\}P\{sick\}}{P\{test+\}}$ = $\frac{0.99 \cdot 0.001}{0.01} = .099$.

The test is not at all useless – it has brought down an individual's likelihood of being sick by orders of magnitude, from one-tenth of one percent to ten percent. But it's still not nearly as accurate as the "99%" label might imply.

Many healthcare providers don't quite get this and explain it merely as "don't be too worried until we do further tests." But this is one reason why broad-based tests can be very expensive and not very helpful. These tests are much more useful if we first narrow down the population of people who might have the disease. For example home pregnancy tests might be 99% accurate but if you randomly selected 1000 people to take the test, you'd find many false positives. Some of those might be guys (!) or women who, for a variety of reasons, are not likely to be pregnant. The test is only useful as one element of a screen that gets progressively finer and finer.

Counting Rules

If A can occur as N_1 events and B can be N_2 events then the sample space is $N_1 \cdot N_2$ (visualize a contingency table with N_1 rows and N_2 columns).

Factorials: If there are N items then they can be arranged in $N! = (n)(n-1)(n-2)\ldots(1) = \prod_{i=0}^{N-1}(N-i)$ ways.

Permutations: **n** events that can occur in **r** items (where order is important) have a total of $nPr = \frac{n!}{(n-r)!}$ possible outcomes.

Combinations: n events that can occur in r items (where order is not important) have $nCr = \frac{n!}{r!(n-r)!}$ possible outcomes – just the permutation divided by r! to take care of the multiple ways of ordering.

So to apply these, consider computer passwords (see NYTimes article below).

The article reports:

> Mr. Herley, working with Dinei Florêncio, also at Microsoft Research, looked at the password policies of 75 Web sites. ... They reported that the sites that allowed relatively weak passwords were busy commercial destinations, including PayPal, Amazon.com and Fidelity Investments. The sites that insisted on very complex passwords were mostly government and university sites. What accounts for the difference? They suggest that "when the voices that advocate for usability are absent or weak, security measures become needlessly restrictive."

Consider the simple mathematics of why a government or university might want complex passwords. How many permutations are possible if passwords are 6 numerical digits? How many if passwords are 6 alphabetic or numeric characters? If the characters are alphabetic, numeric, and fifteen punctuation characters (, . _ - ? ! @ # $ % ^ & * ' ")? What if passwords are 8 characters? If each login attempt takes 1/100 of a second, how many seconds of "brute-force attack" does it take to access the account on average? If there is a penalty of 10 minutes after 3 unsuccessful login attempts, how long would it take to break in? (Of course, the article notes, if password requirements are so arcane that employees put their passwords on a Post-It attached to the monitor, then the calculations above are irrelevant.)

(for fun, here's another example of Joint/Marginal Distributions)

Tiger Mother Amy Chua in WSJ, Jan 8, 2011

A lot of people wonder how Chinese parents raise such stereotypically successful kids. They wonder what these parents do to produce so many math whizzes and music prodigies, what it's like inside the family, and whether they could do it too. Well, I can tell them, because I've done it. Here are some things my daughters, Sophia and Louisa, were never allowed to do:

- *attend a sleepover*
- *have a playdate*
- *be in a school play*
- *complain about not being in a school play*
- *watch TV or play computer games*
- *choose their own extracurricular activities*
- *get any grade less than an A*
- *not be the No. 1 student in every subject except gym and drama*
- *play any instrument other than the piano or violin*
- *not play the piano or violin.*

I'm using the term "Chinese mother" loosely. I know some Korean, Indian, Jamaican, Irish and Ghanaian parents who qualify too. Conversely, I know some mothers of Chinese heritage, almost always born in the West, who are not Chinese mothers, by choice or otherwise. I'm also using the term "Western parents" loosely. Western parents come in all varieties.

So you could go to PUMS and look at first-generation immigrants with parents from China, compare with other first-generation kids, see where are the Tiger Moms…