Lecture Notes Supplement on CES
Economics of the Environment and Natural Resources/Economics of Sustainability
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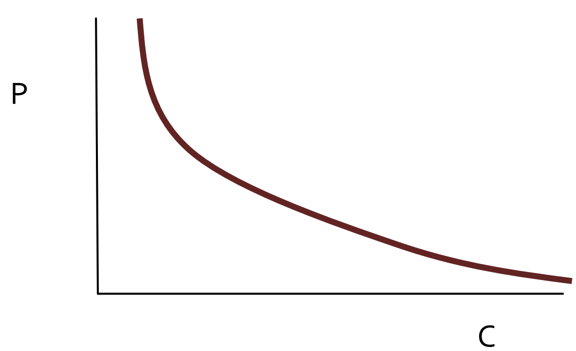
Note on CES utility functions
Some papers use a Constant Elasticity of Substitution (CES) utility function in the economic theory. These are common but used not so much for their realism but to simplify some of the formulas.

A CES utility has a general form of
$$U = \frac{C^{1-\sigma}}{1-\sigma},$$
which looks a bit awful but is "simple" insofar as it makes later formulas so.

Since people make decisions on the margin, we look at marginal utility, which is the derivative,
$$MU = \frac{dU}{dC} = (1-\sigma)\frac{1}{1-\sigma}C^{1-\sigma-1} = C^{-\sigma}.$$

At optimum, C is chosen to make the price, P, equal to Marginal Utility (MU), so
$$P = C^{-\sigma},$$
which is the demand curve. Plot it and note it has a general shape as:



which looks "like a demand curve should."

The elasticity of demand at any point is $\frac{\%\Delta C}{\%\Delta P}=\frac{\frac{\Delta C}{C}}{\frac{\Delta P}{P}}=\left(\frac{P}{C}\right)\frac{1}{\left(\frac{dP}{dC}\right)}$, and we can find $\frac{dP}{dC}$ by taking the derivative of the demand curve, $P=C^{-\sigma}$, $\frac{dP}{dC}=-\sigma C^{-\sigma-1}$. Substitute into the elasticity equation for P and the derivative, to get

$$\left(\frac{P}{C}\right)\frac{1}{\left(\frac{dP}{dC}\right)}=\left(\frac{C^{-\sigma}}{C}\right)\frac{1}{\left(-\sigma C^{-\sigma-1}\right)}=\left(C^{-\sigma-1}\right)\frac{1}{\left(-\sigma C^{-\sigma-1}\right)}=-\frac{1}{\sigma}.$$

A constant elasticity, as promised.