Possible Solutions for Homework 2

Economics of Sustainability

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You are encouraged to form study groups to work on these problems. However each student must hand in a separate assignment: the group can work together to discuss the papers and comment on drafts, but each study group member must write it up herself/himself. Please submit homework assignments on Blackboard.

1. What are the names of people in your study group?
2. Consider some discount rate calculations. You can pick your annualizing frequency but be consistent so that you can compare answers.
    a. If the interest rate is 3%, what is the discounted value of $1000 in 10 years?
    Spreadsheet has calculations. Can calculate exp{-0.03*10}*1000 = 740.82. Or $\frac{1000}{(1+.03)^{10}}$=744.09.
    b. If the interest rate is 1%, what is the discounted value of $1000 in 10 years?
    Either exp{-.01*10}*1000 = 904.84 or $\frac{1000}{(1+.01)^{10}}$=905.29.
    c. If the interest rate is 5%, what is the discounted value of $1000 in 10 years?
    Either exp{-.05*10}*1000 = 606.53 or $\frac{1000}{(1+.05)^{10}}$=613,91. Note how much a relatively small change in the rate can change the present value.
    d. Suppose you are uncertain what interest rate to use, whether 1%, 3%, or 5%. What is the average present discounted value of your calculations? What if you used the average interest rate to do a single discount calculation? Can you figure out the interest rate implied by averaging the discounted values?
    The average interest rate is 3% so that would give the same answer as in (a). But average present value (for continuous compounding) is 750.73. Set this equal to exp(-r*10}*1000 and solve for r to get 2.87%. The average present value for annual discrete compounding is 754.43. Set this equal to $\frac{1000}{(1+r)^{10}}$ and now find r=2.86%. Sometimes you see people present a range of values that they pop out of a spreadsheet – but you have to think about which values you might average since Jensen's Inequality tells us that averaging over nonlinear functions is complicated.
    e. At a discount rate of 2%, what is the present value of 3 payments made in 10, 12, and 15 years in the future, each of $1000?
    In this case it is straightforward to add up the values. The value of one payment in 10 years is exp{-.02*10}*1000; a payment in 12 years is exp{-.02*12}*1000; in 15 years is exp{-.02*15}*1000 – then sum these to get the total value. The discrete case is analogous.
    f. Can calculate effect of changing interest rates.

i. What is discounted value, as of 10 years in the future, of $1000 paid 20 years in the future, when the interest rate between 10 and 20 years is 2%?

This is again exp{-.02*10}*1000 since there is a ten-year horizon, so 818.73 as of 10 years in the future.

ii. What is discounted value of the answer from part (i), at interest rate of 3%, today?

This is exp{-.03*10}*818.73 = 670.32. Alt, $\frac{1000}{(1+.02)^{10}}$ = 820.35, then $\frac{820.35}{(1+.03)^{10}}$.

Again we can back-solve to find what interest rate over the entire 20 years would give the same result – in this case 2.5%. (Sometimes averaging works.)

g. Consider a case where we use (something like) hyperbolic discounting. To value $1m paid in each of the next 100 years, we use a discount rate of 0.5% from years 50-100, a discount rate of 1% for years 25-49, and a discount rate of 2% for years up to 25. What is the discounted value? What would be the discounted value at a flat rate of 1%?

See spreadsheet attached. In this case the answer is different than what you would get from using just the average interest rate of 1%.

3. Suppose a possible gas well site, currently untapped, could produce gas sufficient to turn a profit of $1m in 6 years. How much is the site worth now, if the discount rate were 8%? If the discount rate were 12% because of a greater riskiness? If there is a 50% chance that the rate is 8%, and a 50% chance that the rate is 12%, what is the well worth? What interest rate would give the same value? *(This ignores the uncertainty in gas prices over that time horizon. Assume these are hedged; we'll discuss those possibilities later.)*

At a 8% discount, that profit is (at annual discounting) = $\frac{1{,}000{,}000}{(1+.08)^6}$ = 630,170; at 12% it's 506,631. The average is 568,400 which is what would be implied by a 9.87% interest rate – which is a bit less than the average interest rate.

At continuous discounting the 8% implies $e^{-.08*6}1{,}000{,}000$ = 618,783; 12% implies 486,752. The average value is 552,768 which implies a 9.88% rate, again a bit less than the average rate.