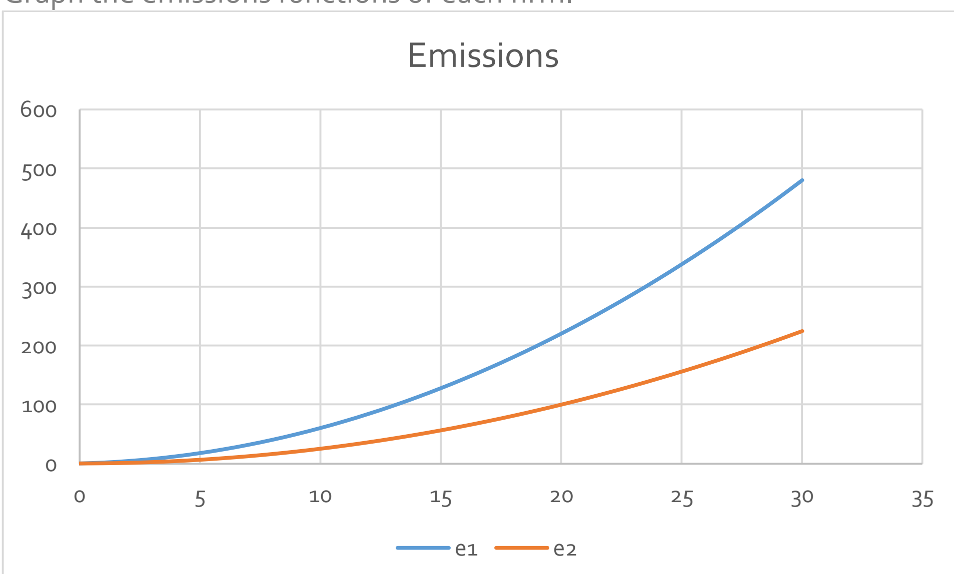
Possible Solutions for Homework 4

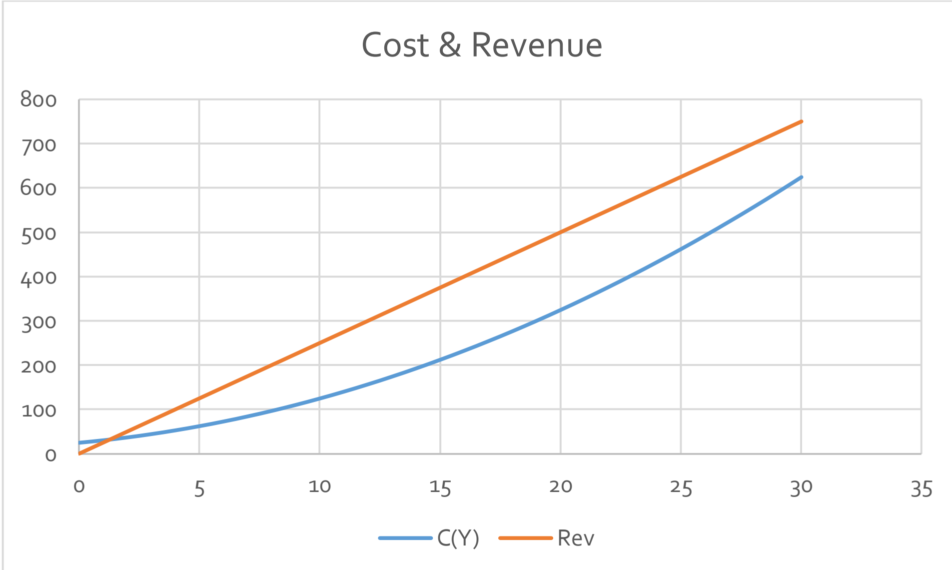
Economics of Sustainability

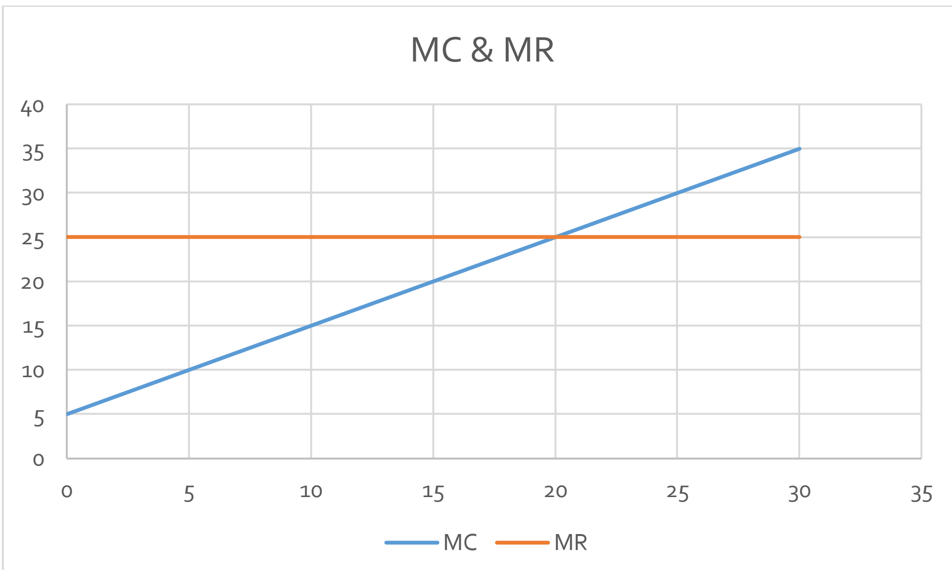
K Foster, Colin Powell School CCNY, Spring 2016

1. What are the names of people in your study group?
2. Can you sketch an argument for how you would respond to economic interpretations of sustainability? Do you agree with them? Where do you see problems with the interpretation? *(Don't write a dissertation, a page or so is fine!)*
3. Give an example (like mine of "2 Guys & a Truck") of a simple production with one or two inputs. *(Clever and/or relevant to sustainability would be even better.)* Do you think it shows diminishing marginal returns of each input individually? What do you think are the returns to scale?
4. Consider regulations of an industry with 2 sorts of plants, designated (with a complete failure of imagination) as type 1 and type 2. Costs for both types of plant are $c(y) = 25 + 5y + \frac{1}{2}y^2$. Type 1 plants are dirtier and produce emissions at a rate of $e_1 = y_1 + \frac{1}{2}y_1^2$; type 2 plants just $e_2 = \frac{1}{4}y_2^2$. Each unit of output, y, is sold for a price of 25.
   a. Graph the emissions functions of each firm.



   b. Create a table of costs, revenue, and profit for different levels of output. Assuming that emissions are free, what level of output would each plant type choose?
   See spreadsheet; each firm would choose Y=20 since they have the same costs.
   c. Add columns to the table for $\frac{\Delta c}{\Delta y}$ and $\frac{\Delta Rev}{\Delta y}$ – either use some calculus to find $\frac{dc}{dy}$ and $\frac{dRev}{dy}$ or just find the differences between integer values of y. What happens around the profit-maximizing level? Graph Costs and Revenues. Separately graph $\frac{\Delta c}{\Delta y}$ and $\frac{\Delta Rev}{\Delta y}$.





Profits are largest where MC=MR, where the change in costs is just identical to change in revenue.

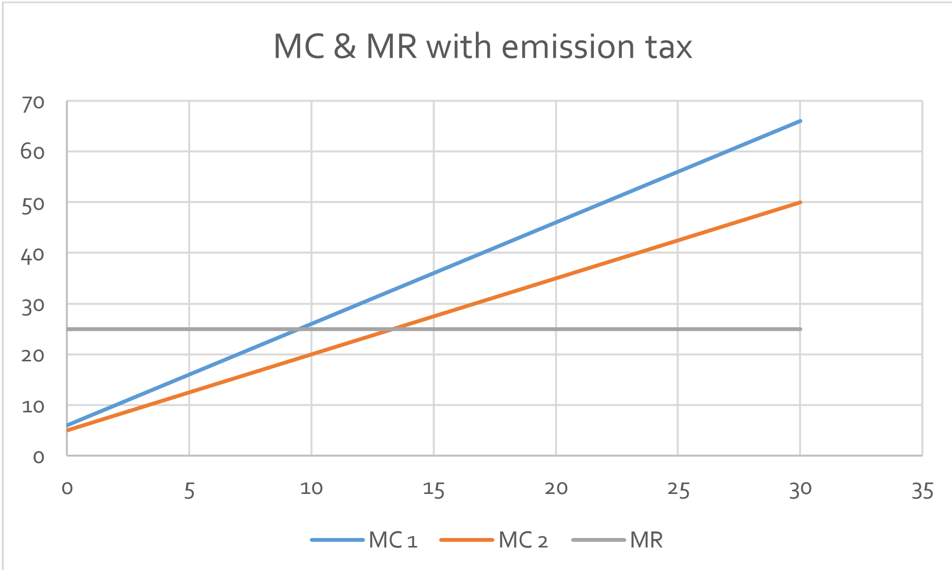
d. Suppose regulations capped plant emissions at 100 – what level of output would the plants choose? Is this efficient – is there a way to produce the same output with fewer emissions? Or could the plants produce more output with the same emissions?

Firm 1 would hit the emissions cap (solve 100=y+.5y² to find y=13.18 from the quadratic formula) so would maximize its profits at that level. Firm 2 is unchanged since its emissions are 100 at the profit-maximizing level. So total emissions E = e1 + e2 = 200 and Y = y1 + y2 = 33.18. But if Firm 1 produced one less unit of output, emissions would decrease by (1+y1), about 14. If Firm 2 simultaneously produced one more unit of output, emissions would increase by .5y2 which is 10. So output could be kept constant while emissions would fall. To minimize emissions with the constraint that Y=y1+y2=33.18 takes a bit of math – see the end.

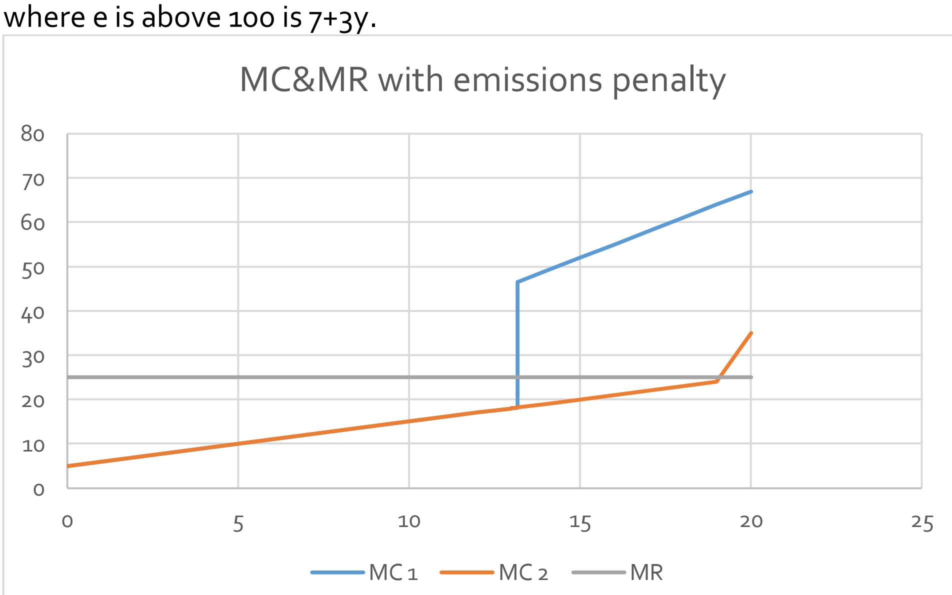
e. Suppose emissions were taxed at a rate of $1 per unit of emission – what would be the new amounts of output chosen at each plant?

Now costs are $c_1(y) = \left(25 + 5y_1 + \frac{1}{2}y_1^2\right) + e_1 = c(y) = \left(25 + 5y_1 + \frac{1}{2}y_1^2\right) + y_1 + \frac{1}{2}y_1^2 =$ $c(y) = 25 + 6y_1 + y_1^2$; $c_2(y) = \left(25 + 5y_2 + \frac{1}{2}y_2^2\right) + e_2 = c_2(y) = \left(25 + 5y_2 + \frac{1}{2}y_2^2\right) + \frac{1}{4}y_2^2 =$

$c_2(y) = 25 + 5y_2 + \frac{3}{4}y_2^2$. Marginal costs for Firm 1 are 6+2y. Marginal costs for Firm 2 are 5+1.5y. Set these equal to 25 so y1 = 9.5 and y2 = 13.33. The dirty firm will reduce output by more than the cleaner firm will.



f. Another way of looking at the emissions cap (each plant's emissions must be 100 of less) is that additional emissions are penalized at a rate of, say, $2 per unit above 100. Does this shift the profit-maximizing choice for each plant?
In this case each firm's cost curve has a jump when e ≥ 100. So for Firm 1 (it's not relevant for 2), cost is as before if e<100 then goes to $\tilde{c_1}(y) = \left(25 + 5y_1 + \frac{1}{2}y_1^2\right) + 2e_1 I\{e_1 \geq 100\}$ where I{ } is an indicator function taking value of 1 if the statement in brackets is true and zero else. For values where it is true, $\tilde{c}(y) = \left(25 + 5y_1 + \frac{1}{2}y_1^2\right) + 2y_1 + y_1^2 = 25 + 7y_1 + \frac{3}{2}y_1^2$. So MC where e is above 100 is 7+3y.



In this case Firm 1 would not produce more than before since the extra cost isn't worth it. However if the price of its output were to rise, the firm might choose to pay the penalties and produce more output.

g. (*Extra*) With a bit of calculus, find the optimal choices for any given emission tax, some level *T*. What is the marginal amount that a plant would be willing to pay for the last unit of emission?

Costs for each firm are $c_1(y) = \left(25 + 5y_1 + \frac{1}{2}y_1^2\right) + Te_1$ and $c_2(y) = \left(25 + 5y_2 + \frac{1}{2}y_2^2\right) + Te_2$.

For Firm 1, $c_1(y) = \left(25 + 5y_1 + \frac{1}{2}y_1^2\right) + T\left(y_1 + \frac{1}{2}y_1^2\right)$ = $c_1(y) = 25 + (5+T)y_1 + \frac{1}{2}(1+T)y_1^2$. Set MC = p so $MC_1(y) = (5+T) + (1+T)y_1 \;\; = p$ so $y_1 = \frac{(p-5-T)}{1+T}$.

For Firm 2, $c_2(y) = \left(25 + 5y_2 + \frac{1}{2}y_2^2\right) + T\left(\frac{1}{4}y_2^2\right)$ = $c_2(y) = 25 + 5y_2 + \frac{1}{2}\left(1 + \frac{1}{2}T\right)y_2^2$. $MC_2(y) = 5 + \left(1 + \frac{1}{2}T\right)y_2 = p;\, y_2 = \frac{p-5}{\left(1+\frac{1}{2}T\right)}$.

h. (*Tricksy*) What if emissions depended on industry output not individual firm choice, so $e_1 = (y_1 + y_2) + \frac{1}{2}(y_1+y_2)^2$ and $e_2 = \frac{1}{4}(y_1+y_2)^2$.
   If emissions by each firm were taxed or regulated, then this would introduce strategic consequences since each firm would have to have expectations about the other firm's strategy and plans for its emissions. (Econ students should be able to find the Nash equilibrium in this case.)

Details of (d): To minimize emissions with the constraint that Y=y1+y2=33.18, set $y_2 = 33.18 - y_1$. Want to $min[e_1 + e_2]$ = $min\left[y_1 + \frac{1}{2}y_1^2 + \frac{1}{4}y_2^2\right]$ = $min\left[y_1 + \frac{1}{2}y_1^2 + \frac{1}{4}(33.18 - y_1)^2\right]$ = $min\left[y_1 + \frac{1}{2}y_1^2 + \frac{1}{4}(33.18^2 - 66.36y_1 + y_1^2)\right]$ = $min\left[y_1 + \frac{1}{2}y_1^2 + \frac{33.18^2}{4} - 16.59y_1 + \frac{1}{4}y_2^2\right]$ = $min\left[-15.59y_1 + \frac{3}{4}y_1^2 + \frac{33.18^2}{4}\right]$. So set the first derivative to zero to minimize, $15.59 = \frac{3}{2}y_1$ and y1=10.39, y2 = 22.79.